On the optimal design of a Financial Stability Fund*

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Abstract

A Financial Stability Fund set by a union of sovereign countries can improve countries' ability to share risks, borrow and lend, with respect to the standard instrument used to smooth fluctuations: sovereign debt financing. Efficiency gains arise from the ability of the fund to offer long-term contingent financial contracts, subject to limited enforcement and moral hazard constraints. In contrast, standard sovereign debt contracts are uncontingent and subject to untimely debt rollovers and default risk. We develop a model of the *Financial Stability Fund* (*FSF*) as a long-term partnership with limited commitment (limited ex-post transfers). We quantitatively compare the constrained-efficient *FSF* economy with the incomplete markets economy with default. In particular, we characterize how (implicit) interest rates and asset holdings differ, as well as how both economies react differently to the same productivity and government expenditure shocks. In our economies, 'calibrated', to the euro area 'euro-stressed countries', there are substantial efficiency gains achieved by establishing a well-designed *Financial Stability Fund*; this is particularly true when economies experience negative shocks. Our theory provides a basis for the design of an *FSF* - for example, for the Eurozone, as recommended by the Four and Five Presidents' reports (2012 and 2015)- and a theoretical framework to assess alternative risk-sharing mechanisms; e.g. the combination of the *European Stability Mechanism* (ESM) conditional interventions and the QE policies of the ECB.

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1 Introduction

> "For all economies to be permanently better off inside the euro area, they also need to be able to share the impact of shocks through risk-sharing within the EMU."

The quote from the *Five Presidents' Report* (2015) recognizes a widely accepted fact: without a Federal Budget, or an institutional framework for the Euro Area with similar fiscal automatic stabilizers, it is unlikely that it will efficiently exploit its capacity for risk-sharing local or country risks with only private risk sharing and the existing EMU institutions[1]. In the aftermath of the financial and euro crises, with the subsequent upsurge of social unrest and discontent, the Five Presidents' call seems timely and urgent. However, almost surprisingly, they leave the task for a future date:

> "In the medium term, as economic structures converge towards the best standards in Europe, public risk-sharing should be enhanced through a mechanism of fiscal stabilisation for the euro area as a whole."

This raises two issues, how such "EMU fiscal capacity with limited asymmetric shock absorption function " (*Four Presidents' Report* (2012)[2]) should be designed and operate, and whether EMU countries need first to converge in order for such an institution to operate efficiently. We address these two issues in this paper. We develop a dynamic model of a *Financial Stability Fund* (*FSF*), as a long-term partnership with two features that are usually seen as the most problematic for a risk-sharing institution to be sustainable. First, risk-sharing transfers should not become persistent, or permanent, transfers beyond the level of redistribution that partners would accept at any point in time; i.e. *ex-post* not only *ex-ante*. Second, as in any insurance contract, the *FSF* must take into account *moral hazard* problems; e.g. by avoiding current political costs, governments may increase future social and economic risks. The mutual consented level of *ex-post* redistribution across countries is affected by how different their risk profiles are; nevertheless, as our model shows, countries can be very diverse and subject to very asymmetric risks and still the *FSF* provide risk-sharing. This, and similar statements, regarding the efficiency of the *FSF* require a quantitative answer and, to this end, we use 1980-2015 data from the euro area ' stressed' countries to see how they would have performed during this period. However, to assess the efficiency of the *FSF* it must be in relation to some other risk-sharing mechanism. We use, as a benchmark, an incomplete markets model where sovereign countries issue long-term defaultable debt in order to smooth their consumption. The choice of defaultable sovereign non-contingent debt is especially appropriate to study how countries react to severe shocks under the two regimes, as a way of comparing how ' stressed' countries would have behaved if they had been subject to shocks similar to the ones they had actually experienced - while using sovereign debt as a smoothing instrument - in the counterfactual of having been part of a much larger, and less risky, *FSF*.

We are not the first ones to address these issues and there are many proposals for how risks could be shared in a monetary union, as there are for how to share the risks and costs of (at least part of)

[1] For example, Furceri and Zdzienicka (2015) estimate that the percentage of non-smoothed GDP shocks is 20% in Germany, 25% in the United States, but 70% in the Euro Area. More recently, Beraja (2016) has done the counterfactual exercise of having the United States being Independent States; using his ' Semi-Structural Methodology'. He finds that, for example, if the employment rate's cross-state standard deviation was 2.6% in 2010, it would had been 3.5% if it had not been a fiscal union, a difference of the same magnitude than the "Great Moderation".

[2] Also known as *The Van Rompuy Report.*

the existing debt liabilities. For example, as an implicit criticism of different proposals to issue some form of joint-liability eurobonds, Tirole (2015) emphasises the asymmetry issue: the optimal (one-period) risk-sharing contract with two symmetric countries is a joint liability debt contract serving as a risk-sharing mechanism, while the optimal contract between two countries with very different distress probabilities is a debt contract with a cap and no joint liability, where the cap depends on the extent of solidarity, which is given by the externality cost of debt default on the lender. This can be seen as giving theoretical support to the Five Presidents' argument in favour of postponing the implementation of a euro area *FSF* to achieve more symmetry. However, in a long-term partnership, unconditional debt contracts with default are not efficient (even if it is long-term unconditional debt) and, in particular, as our model shows, even if countries face asymmetric risks, risk-sharing can be very substantial within an *FSF*, provided the limits to *ex-post* redistribution are not too stringent.

On the more practical side, a positive development within the euro crisis has been the creation of the *European Stability Mechanism* in 2012, which treaty (Ch. 4 Art. 12.1) establishes as its first principle that:

> If indispensable to safeguard the financial stability of the euro area as a whole and of its Member States, the ESM may provide stability support to an ESM Member subject to strict conditionality, appropriate to the financial assistance instrument chosen.

While this first principle assesses the need for contingent fund contracts, it also limits its funding to ' extreme' events. Conditionality is a property of the optimal fund contract that we characterize, but in contrast with the *ESM*, the *FSF* is designed as a risk-sharing mechanism, which in principle should account for a much broader set of risk-events. The latter is important, not only because it enhances the risk-sharing possibilities but also because commitment to normal transfers builds trust among participating partners, in contrast with ' rescue mechanisms' that tend to create a stigma on those who need to participate in its schemes and, in fact, often participate in them in situations that could have been avoided with a better risk-sharing arrangement.

Our model of the *FSF* as a partnership builds on the literature on dynamic optimal contracts with enforcement constraints (e.g. Marcet and Marimon 1999, 2016), as well as on the related literature on price decentralization of optimal contracts (e.g. Alvarez and Jermann 2000, Krueger et al. 2008). Our benchmark incomplete markets economy with long-term debt with Default, builds on the model of Chatterjee and Eyigungor (2012) who extend the sovereign default models of Eaton and Gersovitz (1981) and Arellano (2008) to long-term debt.

In order to properly compare the *FSF* economy with the incomplete markets economy, we 'decentralize' the fund contract generating the appropriate prices. For example, both in the *incomplete markets* economy with default, and in the two-sided limited *FSF* economy interest rates may differ from the risk-free rate. In the former, the *positive spreads* reflect the risk of default, while in the latter the *negative spreads* reflect the risk that the lender's participation constraint becomes binding. Lower interest rates deter the lender from lending, implementing the *FSF* lender's enforcement constraint. Default results in autarky, with a small likelihood of returning to the fund. In the *incomplete markets* economy, voluntary default occurs when the cost of repaying the debt is higher than the cost of getting into autarky, with a low probability of a return to the bond market. In contrast, in an *FSF* economy, countries do not renege on the fund contract in equilibrium.

Although for simplicity we analyze the polar case here, the *FSF* could also be a complementary mechanism to debt financing[3]. In this context a main advantage for countries to participate in the fund is that their ? possibly non-sustainable - short-term and non state-contingent debt is transformed into a state-contingent long-term debt. In this sense, the fund provides a technology of maturity transformation. In our model, the fund, acting as a risk-neutral lender, can freely borrow and lend in the international markets at a risk-free interest rate, while providing long-term conditional financing to the borrower.

Our economies are subject to technology and government expenditure shock processes. The processes are modeled as Markov processes calibrated to the euro area ‘ stressed countries’ (AMECO 1980 - 2015). The former is exogenous, while the stochastic behaviour of government expenditure shocks can be affected by government behaviour; e.g. lower government expenditures are more likely if the government implements policies which are (politically) costly to it. One can argue that there should not be risk-sharing of government expenditures. However, we use government expenditures as a proxy for government liabilities that, in practice, may result in higher consumption volatility without some form of risk-sharing or consumption smoothing. Computing our economies allows for close inspection of the policy functions, showing how different regimes result in different consumption, labour and asset holdings decisions, and in particular, how the same sequence of productivity and government shocks affects agents' decisions differently, resulting in different equilibrium paths across regimes. These differences in policies also translate, in our computed economies, into substantial welfare gains through implementing a properly designed *FSF*.

We also show how these economies react differently to a permanent, as well as to a transitory shock. Our economies with relatively more impatient risk-averse agents (countries), with separable disutility for labour, contracting with risk-neutral agents, have clear, and known, efficiency benchmarks: consumption of the borrowing country should decay smoothly and effort should be positively correlated with productivity shocks (higher effort when it is more productive). Economies with limited enforcement constraints can be relatively close to such benchmarks, except that enforcement constraints deter from consumption decay or may force agents to work harder in low productivity states. Nevertheless, distortions are typically larger in the economies with incomplete markets; particularly so when default risk is high. Our computed exercises provide clear images of these differences. For example, of how an *FSF* economy provides better financing opportunities to the borrower at the same cost to the lender, or copes better with shocks that result in default in the incomplete markets economy. Crises amplify the gains from establishing an *FSF*, as opposed to simply relying on sovereign debt.

2 The economy and the benchmark case of sovereign debt financing

We consider a standard infinite-horizon representative agent economy, where the agent has preferences for current leisure, $l = 1 - n$, and consumption, c and effort, e, represented by $U(c, n, e) := u(c) + h(1-n) - v(e)$ and discounts the future at the rate β. We make standard assumptions on preferences[4]. The agent has access to a decreasing returns labor technology $y = \theta f(n)$, where $f' > 0$, $f'' < 0$ and θ is a productivity shock, assumed to be Markovian; $\theta \in (\theta_1, ..., \theta_N)$, $\theta_i < \theta_{i+1}$. The economy is a small open economy in a world with no uncertainty with interest rate r satisfying $1/(1 + r) \geq \beta$;

[3]Central bank sovereign bond market interventions, such as the ECB interventions in the euro crisis, also complement normal debt financing from households.

[4]In particular, we assume that $(c, n, e) \in \mathbb{R}^3_+, n \leq 1$, and u, h, v are differentiable, with $u'(c) > 0, u''(c) < 0, h'(c) > 0, h''(c) < 0$ and $v'(c) > 0, v''(c) > 0$.

an inequality that, in general, we will assume to be strict. In order to borrow and save, the agent, which we also identify with the government of the country, may have access to different financial technologies, which will define different regimes, which we also call different economies.

The country also faces government expenditure shocks $G = G^c + G^d$, which together with the productivity shock defines the exogenous state, denoted by $s = (\theta, G)$. G^c takes discrete values from $G^c \in \{G_1^c, \ldots, G_{N_G}^c\}$ and is a Markov process with transition probability $\pi^{G^c}(G'|s,e)$, and G^d is i.i.d. over time with continuous distribution ν over $G^d = \left[-\bar{m}, \bar{m}\right]$. In addition, G^c and G^d are independent with each other. The interpretation is that G^c are government expenditures and the distribution of next period expenditures depend on the policies that the government implements in the current period. In particular, the government can have a a better distribution of tomorrow's expenditures if exercises sufficient effort in the current period (e.g. politically costly reforms are more likely to result in lower government expenditures). G^d is a residual shock that cannot be affected by government actions[5]. More precisely, we assume that given the current state, $s = (\theta, G)$, the next period realizations of θ and G are independent and only the latter depends on effort. That is

$$\pi(s'|s,e) = \pi^{\theta}(\theta'|s)\pi^{G}(G'|s,e).$$

We assume that the cost of this effort are expressed in utility terms given by $v(e)$. We assume that high effort increases the probability of lower government expenditure, in this sense we can think about effort as ' austerity' measures with utility costs which reduce primary deficit. We assume that both $v(\cdot)$ and $\pi^G(G'|s,e)$ are continuous and twice differentiable in effort, moreover we assume that $v(\cdot)$ is convex.

2.1 The incomplete market model with long-term bond financing

The incomplete market model is a quantitative version of the seminal model by Eaton and Gersovitz (1981). We integrate three modeling advances in the recent literature, namely endogenous labor and output, long-term bond, and asymmetric default penalty, to achieve a more complete description of the business cycle dynamics of an small open economy with sovereign debt. We detail the specification of the baseline incomplete market model in this section.

In the incomplete market model, the borrower can issue or purchase *long-term* bonds, which promise to pay constant cash flows across different states. We model the long-term bond in the same way as Chatterjee and Eyigungor (2012).

A unit of long-term bond is parameterized by (δ, κ), where δ is the probability of continuing to pay out the coupon in the current period, and κ is the coupon rate. Alternatively, $1 - \delta$ is to the probability of maturing in the current period, and this event is independent over time. The size of each bond is infinitesimal and the payment of each bond is independent in cross-section. As a result, on average one unit of bond (δ, κ) will repay $(1 - \delta) + \delta\kappa$ in the current period for sure. It also follows that the bond portfolio has a recursive structure, in which only the size of total outstanding debt b matters, regardless when a particular issue of the bond enters into the portfolio. Moreover, δ directly captures the duration of the bond: if $\delta = 0$ the bond becomes the standard one-period debt, and in general, the average maturity of the bond equals to $1/(1 - \delta)$, which is increasing in δ. Coupon rate

[5] The introduction of G^d is for technical reasons, as in Chatterjee and Eyigungor (2012). Notice that the composite G shock admits a Markov structure as well, with state space $G = \cup_i \left[G_i^c - \bar{m}, G_i^c + \bar{m}\right] \subset R$ and transition kernel $\pi^G = \pi^{G_c} \otimes \nu$.

κ provides a flexible way to capture the coupon payment: $\delta\kappa$ equals to the coupon payment on each unit principal of outstanding debt.

For an outstanding bond portfolio of size b, its cash flow stream is given by $(1-\delta)b + \delta\kappa b, \quad \delta(1-\delta)b + \delta^2\kappa b, \quad \ldots$. When there is no default, the price of a unit of a *riskless* long-term bond (δ, κ), given a constant discount rate r, is:

$$\mathsf{q} = \sum_{t=0}^{\infty}[(1-\delta) + \delta\kappa]\frac{\delta^t}{(1+r)^{t+1}} = \frac{(1-\delta)+\delta\kappa}{r+1-\delta}.$$

2.2 The budget Constraint and default

Let b_t denote the size of the bond portfolio (δ, κ) held by the borrower at the beginning of time t. Following the convention in the literature, $b_t \geq 0$ means holding assets while $b_t < 0$ means having debt. The borrower first makes a decision on whether to default on the promised bond payment of the entire bond portfolio b_t.

No default When the borrower chooses not to default, then the bond payment $(1-\delta)b_t + \delta\kappa b_t$ will be settled as promised: if $b_t \geq 0$, then the bond payment is part of the borrower's time t income; else if $b_t < 0$, then the borrower will make the required payment to the lender. Choosing not to default allows the borrower to stay in the bond market, so that the borrower may choose the bond holding position b_{t+1} for the next period. The difference between b_{t+1} and the remaining principal δb_t is the net issuance at time t. Due to the recursive structure of the long-term bond, the cash flows starting from $t+1$ onward of both b_{t+1} and δb_t are proportional, and therefore the same unit bond price applies to both. As to be explained below, the bond price is a function of the exogenous shock s_t and the bond position b_{t+1} for the next period, thus we use $q(s_t, b_{t+1})$ to denote this function. It follows that when the borrower chooses not to default, the budget constraint is as follows:

$$c_t + q(s_t, b_{t+1})(b_{t+1} - \delta b_t) \leq \theta_t n_t^\alpha - G_t + (1-\delta+\delta\kappa)b_t.$$

Default Upon choosing default, the borrower is excluded from the bond market immediately and enters into autarky. As a result, the time t consumption is given by

$$c_t = \theta_t n_t^\alpha - G_t.$$

The exclusion lasts for a random number of periods. If the borrower is excluded from the market in the previous period, then with probability $\lambda < 1$ the borrower regains access to the bond market in the current period, and with remaining probability $1-\lambda > 0$ the borrower stays in autarky. Moreover, upon regaining access to the bond market, the borrower starts from a zero bond position.

Besides the exclusion from the bond market, the borrower also suffers from a productivity penalty in autarky. As in Arellano (2008), the penalty takes the following form:

$$\theta^p = \begin{cases} \bar{\theta}, & \theta_t \geq \bar{\theta} \\ \theta, & \theta < \bar{\theta} \end{cases} \quad \text{with } \bar{\theta} = \psi\mathbb{E}\theta,$$

which is *asymmetric* in the sense that the magnitude of penalty is zero for low productivity state while equals to $\theta - \bar{\theta}$—increasing in θ—for high productivity state. The level of the penalty is parameterized by $\psi > 0$. Given that $0 < \theta_1 < \cdots < \theta_{N_\theta}$, on the one hand the penalty becomes a benefit if $\psi \geq \theta_{N_\theta}/\mathbb{E}\theta$; and on the other hand, the penalty ceases to be effective if $\psi < \theta_1/\mathbb{E}\theta$, since the borrower can always choose to have zero debt while enjoying higher productivity levels. Asymmetric penalty is crucial for the quantitative performance of models with sovereign debt and default. When the penalty is properly specified, it creates incentive for the borrower to borrow more in good states while deterring default temptation by harsh punishment, and these high levels of debt then induce the borrower to choose default in bad states where the penalty is zero.

2.3 The recursive formulation and bond prices

If b the size of the long-term bond portfolio held by the borrower at the beginning of a period[6], then (s, b), $s = (\theta, G)$, is the state. Let $V_n^{bi}(b,\, s)$ denote the value function of the borrower, in the incomplete market economy, at the beginning of a period before any decisions are made. The value function when the borrower chooses no to default satisfies

$$\begin{aligned} V_n^{bi}(b,\, s) &= \max_{c,n,e,b'} \left\{ U(c, n, e) + \beta \mathbb{E}\left[V^{bi}(b',\, s') \mid s, e \right] \right\} \\ \text{s.t.} \quad & c + G + q(s, b, b')(b' - \delta b) \leq \theta f\,(n) + (1 - \delta + \delta\kappa) b, \end{aligned} \tag{1}$$

where, taking into account that default can occur next period,

$$V^{bi}(b,\, s) = \max\{V_n^{bi}(b,\, s), V^{ai}(s)\}, \tag{2}$$

and $V^{ai}(s)$ is the value of being in autarky, given by

$$\begin{aligned} V^{ai}(\, s) = \max_{n,e} \{ & u\,(\theta^p(\theta) f\,(n) - G) + h(1 - n) - v(e) \\ & + \beta \mathbb{E}\left[\left(1 - \lambda^i\right) V^{ai}(s') + \lambda^i V^{bi}\,(0,\, s') \mid s, e \right], \end{aligned} \tag{3}$$

where λ^i is the probability to come back to the market and be able to borrow.

The reason the bond price $q(s, b, b')$ depends on b is that e, which it is assumed that it is not observable or contractable, affects the distribution of $G^{c\prime}$. The bond price has also a recursive structure. Let the default decision be given by

$$D(\, s, b) = 1 \text{ if } V^{ai}(\, s) > V_n^{bi}(b,\, s) \text{ and } 0 \text{ otherwise;}$$

therefore, the expected default rate is $d(s, b, b') = \mathbb{E}\left[D(s',\, b') \mid\, s, e^*(s, b)\right]$ The equilibrium bond pricing function $q(s, b, b')$ satisfies the following recursive equation:

$$q(s, b, b') = \frac{\mathbb{E}\left[\left(1 - D(s', b')\right)(1 - \delta) + \delta\left[\kappa + q(s',\, b', b''(s',\, b'))\right] \mid\, s, e^*(s, b) \right]}{1 + r},$$

which can also be expressed as:

$$\begin{aligned} q(s, b, b') \quad &= \quad \frac{(1 - \delta) + \delta\kappa}{1 + r}\left(1 - d(s, b, b')\right) \\ &+ \quad \delta \frac{\mathbb{E}\left[\left(1 - D(s',\, b')\right) q(s',\, b', b''(s',\, b') \mid\, s, e^*(s, b) \right]}{1 + r}, \end{aligned} \tag{4}$$

[6] We assume that $b \in \mathcal{B} = [b_{\min}, b_{\max}]$, with $-\infty < b_{\min} < 0 \leq b_{\max} < \infty$., where we will choose $b_{\min}$ and $b_{\max}$ so that in equilibrium the bounds are not binding.

which for the one-period bond, $\delta = 0$, reduces to $q(s,b,b') = \frac{1-d(s,b,b')}{1+r}$. The corresponding interest rate of the long-term bond is given by

$$r^i(s,b,b') = \frac{(1-\delta)+\delta\kappa}{q(s,b,b')} - (1-\delta),$$

resulting in a *positive spread* $r^i(s,b,b') - r \geq 0$, which is strictly positive if $d(s,b,b') > 0$.

In order to keep track of debt flows, it is useful to define the *primary surplus* – or *primary deficit* if negative – which is given by

$$q(s,b,b')(b'-\delta b) - (1-\delta+\delta\kappa)b = \theta f(n) - (c+G)$$

2.4 The effort decision

The optimal policies $c^b(b,s)$, $n^b(b,s)$, $b'(b,s)$ and $n^a(s)$ are standard dynamic programming solutions to (1) and (3), respectively. The effort policy function when there is no default in state $s = (\theta, G)$, $e^b(b,s)$, is given by

$$v'(e) = \sum_{s'} \pi^\theta(\theta'|\theta) \frac{\partial \pi^G(G'|G,e)}{\partial e} V^{bi}(b', s'),$$

where b' is the optimal choice of debt in (1). Similarly, the effort policy function when there is default in state $s = (\theta, G)$, $e^a(s)$, is given by

$$v'(e) = \sum_{s'} \pi^\theta(\theta'|\theta) \frac{\partial \pi^G(G'|G,e)}{\partial e} \left[(1-\lambda)V^{ai}(s') + \lambda V_b^{bi}(0,s')\right],$$

since in (3) the choice of debt is predetermined to be zero[7].

3 The *Financial Stability Fund* as a long-term contract

An economy with a *Financial Stability Fund* (FSF) is modeled as a long-term contract between a fund, or FSF, manager (also called lender), who can freely borrow and lend in the international market, and an individual partner (also called country or borrower), who is 'the representative agent' of the small open economy. We assume that the manager cannot observe the effort of the partner, which implies that the long term contract will have to provide sufficient incentives for the country to implement a (constrained) efficient level of effort. In the *fund contract*, the country consumes c and the resulting transfer to the FSF manager is $\tau = \theta f(n) - (c+G)$; i.e. when $\tau < 0$ the country is effectively borrowing. We consider that there is two-sided limited enforcement; that is, both the FSF manager and the lender can renege of their contract and pursue their outside options at any time-state.

[7] In both cases we have assumed that the effort decision is interior, as it is the case when we parameterise the corresponding functions. Similarly, we provide more structure by assuming that, given current government liabilities G^c, there are two possible distributions of tomorrow's, $\pi^b(\cdot|G^c)$ and $\pi^g(\cdot|G^c)$, and $\pi^g(\cdot|G^c)$ first-order stochastically dominates $\pi^b(\cdot|G^c)$ for all G: there is $\zeta(e)$ with $\zeta'(e) < 0$, such that $\pi^G(G'|G^c,e) = \zeta(e)\pi^b(G'|G^c) + (1-\zeta(e))\pi^g(G'|G^c)$; that is,

$$\frac{\partial \pi^G(G'|G,e)}{\partial e} = -\zeta'(e)\left[\pi^g(G'|G) - \pi^b(G'|G)\right].$$

In state $s^t = (s_0, \ldots, s_t)$, the outside value of the country is the value of autarky, $V^{af}(s^t)$, which is defined as (3) with a probability λ^f of starting a new fund contract. The outside option of the lender is $Z \leq 0$, at any s^t[8], which is determined by the willingness of the FSF (the lender) to accept some level of redististribution or to avoid that the country breaks away. Whether it is *e*x-post altruisitic or self-interested – as in Tirole (2015) – *solidarity*, Z is an important parameter when assessing the efficiency gains of establishing a FSF; in particular, if $Z = 0$ the FSF may still be superior to other mechanisms, since it can still provide some level of risk-sharing and for the impatient borrower can always be a better 'borrowing mechanism'. In the next Section we show how Z constraints the paths of FSF transfers and its effect on prices.

With *two-sided limited enforcement*, denoted (2S), an optimal fund contract is a solution to the following problem:

$$\max_{\{c(s^t),n(s^t),e(s^t)\}} \mathbb{E}\Bigg[\mu_{b,0}\sum_{t=0}^{\infty}\beta^t\left[u(c(s^t))+U(1-n(s^t))-v(e(s^t))\right]$$
$$+\mu_{l,0}\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t\left[\theta(s^t)f\left(n(s^t)\right)-G(s^t)-c(s^t)\right] \mid s_0, \{e\left(s^t\right)\}_{t=0}^{\infty}\Bigg]$$

s.t.

$$\mathbb{E}\left[\sum_{r=t}\beta^{r-t}\left[u(c(s^r))+U(1-n(s^r))-v(e(s^r))\right] \mid s^t, \{e\left(s^r\right)\}_{r=t}^{\infty}\right] \geq V^{af}\left(s_t\right) \tag{5}$$

$$\mathbb{E}\left[\sum_{r=t}\left(\frac{1}{1+r}\right)^{r-t}\theta(s^t)f\left(n(s^t)\right)-c(s^t)-G(s^t) \mid s^t, \{e\left(s^r\right)\}_{r=t}^{\infty}\right] \geq Z, \tag{6}$$

$$\text{and} \qquad v'(e(s^t)) = \sum_{s^{t+1}|s^t}\frac{\partial\pi(s^{t+1}|s_t,\, e(s^t))}{\partial e(s^t)}\tilde{V}^{bf}(s^{t+1}) \tag{7}$$

where the first two constraints (5) and (6) are the intertemporal participation constraints for the borrower and the lender, respectively, and (μ_{b0}, μ_{l0}) are initial Pareto weights. Here the notation is explicit about the fact that expectations are conditional on the implemented effort sequence as it affects the distribution of the shocks. The last constraint (7) is the incentive compatibility constraint with respect to effort[9], where $\tilde{V}^{bf}(s^{t+1})$ is the value of the *FSF* contract to the borrower in state s^{t+1}.

By imposing equality in (7) we have implicitly assumed that effort is interior, that is $e > 0$. The interpretation of this constraint is standard: the marginal cost of increasing effort has to be equal to the marginal benefit. The latter is measured as the change in life-time utility due to the change in the distribution of future shocks as a result of the increasing effort. Note that $\tilde{V}^{bf}(s^{t+1})$ can also be written explicitly as the continuation life-time utilities of the borrower for all continuation states from

[8]Our characterisation easily generalises to the case that the outside value of the manager (lender) is time-state dependent.

[9]Note that, we have used the first-order condition approach here, that is we have replaced by the agent's full optimization problem by its necessary first-order conditions of optimality. According the results of Rogerson (1988), the first-order conditions are also sufficient if the $\pi^G(G'|s,e)$ functions satisfy the monotone likelihood ratio and the convex distribution function conditions described below.

MLR. The probability shifting function $\pi^G\left(G' \mid s,e\right)$ has the *monotone likelihood ratio* property if, for each $e \geq 0$ and s, the ratio $\frac{\partial\pi^G(G'|s,e)/\partial e}{\pi^G(G'|s,e)}$ is non-increasing in G'.

CDF. The functions $\pi^G(G'|s,e)$ satisfy the convex distribution function condition if $\frac{\partial\partial F_{\tilde{G}}(s,e)}{\partial e\partial e}$ is non-negative for every e, s and $\tilde{G}$ where $(F_{\tilde{G}}(s,e) = \sum_{G'\leq\tilde{G}}\pi^G(G'|s,e)$.

next period on. In particular, (7) can also be written as:

$$v'(e(s^t)) = \beta \sum_{s^{t+1}|s^t} \frac{\partial\pi(s^{t+1}|s_t,\, e(s^t))}{\partial e(s^t)} \mathbb{E}\left[\sum_{r=t+1}^{\infty} \beta^{r-(t+1)} \left[U(c(s^r),\, n(s^r),\, e(s^r))\right] \mid s^{t+1}\right].$$

It is known from Marcet and Marimon (1999, 2016) and Mele (2013) that we can rewrite the general fund contract problem as:

$$\begin{aligned}
\min_{\{\gamma_{b,t},\gamma_{l,t}\,\xi_t\}} \max_{\{c_t,n_t,e_t\}} \mathbb{E}\Bigg[\sum_{t=0}^{\infty} \beta^t \big(\mu_{b,t}\left[U(c_t,\, n_t,\, e_t)\right] - \xi_t v'(e_t) \\
+ \gamma_{b,t}\Big[U(c_t,\, n_t,\, e_t) - V^{af}(s_t)\Big]\Big) \\
+ \sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t \left(\mu_{l,t+1}\left[\theta_t f(n_t) - G_t - c_t\right] - \gamma_{l,t} Z\right) \mid s_0\Bigg]
\end{aligned}$$

$$\mu_{b,t+1} = \mu_{b,t} + \gamma_{b,t} + \xi_t \frac{\partial\pi(s_{t+1}|s_t, e_t)/\partial e}{\pi(s_{t+1}|s_t, e_t)}, \quad \text{with } \mu_{b,0} \text{ given, and}$$

$$\mu_{l,t+1} = \mu_{l,t} + \gamma_{l,t}, \quad \text{with } \mu_{l,0} \text{ given.}$$

Here $\beta^t \Pr(s^t)\gamma_b\,(s^t)$, $\beta^t \Pr(s^t)\gamma_l\,(s^t)$ and $\beta^t \Pr(s^t)\xi\,(s^t)$ are the Lagrange multipliers of the limited enforcement constraints (5), (6) and incentive compatibility constraint (7), respectively, in state s^t. That is, with one-sided limited commitment $\gamma_l\,(s^t) = 0$, $\forall t \geq 0$. Notice that, by construction, we have $\mu_{b,t+1}(s_{t+1} \mid s_t)$; that is, with the incentive compatibility constraint (7), the co-state $\mu_{b,t+1}$ is a vector, while without (7) is a number.

We will use a convenient **normalization**, in order to minimise the dimension of the co-state vector. Let $\eta \equiv \beta(1+r) \leq 1$ and normalize multipliers: $v_{i,t} = \gamma_{i,t}/\mu_{i,t}$, $i = b, l$, and

$$\varphi_{t+1} = \frac{\xi_t}{\mu_{b,t}} \frac{\partial\pi(s_{t+1}|s_t, e_t)/\partial e}{\pi(s_{t+1}|s_t, e_t)};$$

then, a new co-state vector is recursively defined as:

$$x_0 = \mu_{l,0}/\mu_{b,0} \text{ and } x_{t+1} = \frac{1 + v_{l,t}}{1 + v_{b,t} + \varphi_{t+1}} \frac{x_t}{\eta}$$

With this normalization, $v_{b,t}$ and $v_{l,t}$ become the multipliers of the limited enforcement constraints, corresponding to (5) and (6), and φ_t the multiplier of the incentive compatibility constraint, corresponding to (7). Similarly, the policy functions defining the *FSF* contract take the form: $c(x,s)$, $n(x,s)$, $e(x,s)$, $\tau(x,s)$ and $v_b(x,s)$, $v_l(x,s)$, $\varphi(x,s)$, satisfying

$$u'(c(x,s)) = \frac{1 + v_l(x,s)}{1 + v_b(x,s)} \frac{x}{\eta} \quad \text{and} \quad \frac{h'(1 - n(x,s))}{u'(c(x,s))} = \theta f'(n(x,s)).$$

The value function of the *FSF* contract takes the form:

$$FV(x,s) = xV^{lf}(x,s) + V^{bf}(x,s); \text{ where,}$$

$$V^{bf}(x,s) = U(c(x,s),\, n(x,s)\, e(x,s)) + \beta\mathbb{E}\left[V^{bf}(x',s') \mid s\right]$$

and

$$V^{lf}(x,s) = \tau(x,s) + \frac{1}{1+r}\mathbb{E}\left[V^{lf}(x',s') \mid s\right];$$

furthermore, $V^{bf}(x,s) \geq V^{af}(s)$, with equality if $v_b(x,s) > 0$ and, similarly, $V^{lf}(x,s) \geq Z$ with equality if $v_l(x,s) > 0$.

4 Decentralization of the fund contract

We now show how to decentralize the optimal contract as a competitive equilibrium with endogenous borrowing constraints, which will allow us to compare the different fund contracts with the debt contract of the economy with incomplete markets. We build on the work of Alvarez and Jermann (2000) and Krueger, Lustig and Perri (2008).

4.1 The competitive equilibrium

In the market equilibrium, the borrower has a home technology that produces $\theta(s^t)f\left(n(s^t)\right)$ with his own labor. The borrower has access to a complete set of one period Arrow securities and solves the following dynamic programming problem:

$$
\begin{aligned}
W^b(a,\, s) &= \max_{(c,\, n,\, e,\, a'(s'))} \left\{U(c,\, n,\, e) + \beta\mathbb{E}\left[W^b(a', s') \mid s\right]\right\} \\
\text{s.t. } & c + \sum_{s'|s} q(s' \mid s)a'(s') \leq \theta(s)f(n) - G(s) + a \\
& a'(s') \geq A_b\left(s'\right) \\
& v'(e) = \beta \sum_{s'|s} \frac{\partial \pi(s'|s,\, e)}{\partial e} W^b\left(\, a'(s'),\, s'\right),
\end{aligned}
$$

where $q\left(s'|s\right)$ is the price of the one period state contingent claim and $a_b\left(s'\right)$ is the amount of state contingent claims chosen by the borrower and $A_b\left(s'\right)$ is an endogenous borrowing limit defined below. The Euler and transversality conditions imply that:

$$
q\left(s^{t+1}|s^t\right) \geq \beta^t \pi\left(s_{t+1}|s_t\right) \frac{u'\left(c\left(s^{t+1}\right)\right)}{u'\left(c\left(s^t\right)\right)}
$$

with equality if $a_b\left(s^{t+1}\right) > A_b\left(s^{t+1}\right)$ and

$$
\lim_{t\to\infty} \sum_{s^t} \beta^t \pi\left(s^t\right) u'\left(c\left(s^t\right)\right)\left[a_b\left(s^t\right) - A_b\left(s^t\right)\right] \leq 0
$$

The lender also has access to a complete set of Arrow securities and similarly solves:

$$
\begin{aligned}
W^l(a,\, s) &= \max_{(c,\, a'(s'))} \left\{c + \frac{1}{1+r}\mathbb{E}\left[W^l(a', s') \mid s\right]\right\} \\
\text{s.t. } & c + \sum_{s'|s} q\left(s'|s\right) a\left(s'\right) = a(s) \\
& a'\left(s'\right) \geq A_l\left(s'\right)
\end{aligned}
$$

As above, the Euler and transversality conditions imply:

$$
q\left(s^{t+1}|s^t\right) \geq \left(\frac{1}{1+r}\right)^t \pi\left(s_{t+1}|s_t\right)
$$

with equality if $a_l\left(s^{t+1}\right) > A_l\left(s^{t+1}\right)$ and

$$\lim_{t\to\infty}\sum_{s^t}\left(\frac{1}{1+r}\right)^t \pi\left(s^t\right)\left[a_l\left(s^t\right)-A_l\left(s^t\right)\right]\leq 0$$

Market clearing implies that:

$$\begin{aligned} c\left(s^t\right)+c_l\left(s^t\right) &= \theta(s^t)f\left(n(s^t)\right)-G\left(s^t\right) \text{ for all } s^t \\ a_b\left(s^{t+1}\right)+a_l\left(s^{t+1}\right) &= 0 \text{ for all } s^{t+1} \end{aligned}$$

We assume that the borrowing limits are properly tight in the sense that satisfy:

$$W^b(A_b\left(s^t\right),\theta(s^t)) = V^{af}(s^t) \tag{8}$$

$$W^l(A_l\left(s^t\right),\theta(s^t)) = Z \tag{9}$$

We conclude our characterization of competitive equilibria, with Arrow-securities and endogenous borrowing limits, by restricting them to those with allocations satisfying the *high implied interest rate condition*, namely:

$$\sum_{t=0}^{\infty}\sum_{s^t}Q\left(s^t|s_0\right)\left[c\left(s^t\right)+c_l\left(s^t\right)\right]<\infty,$$

where present value prices are defined as:

$$Q\left(s^t|s_0\right)=q\left(s^1|s_0\right)q\left(s^2|s^1\right)...q\left(s^t|s^{t-1}\right).$$

4.2 Decentralization

Now we show how a *FSF* contract can be decentralized as a competitive equilibrium with Arrow-securities and endogenous borrowing limits. The interest of doing so is twofold. First, to obtain asset prices supporting the contract, which will allow us to compare them to the asset prices of the incomplete markets economy. Second, to provide a benchmark to other risk-sharing contracts, such as contingent debt contracts. The exercise is standard but nevertheless it reveals an interesting feature: the presence of *negative spreads*!

Let $\{c^*\left(s^t\right), n^*\left(s^t\right), e^*\left(s^t\right), \tau^*\left(s^t\right)\}_{t=0}^{\infty}$ be the allocations of an optimal fund contract. We first, we use the allocations to define the Arrow security prices as follows:

$$q^*\left(s^{t+1}|s^t\right) = \beta\pi\left(s_{t+1}|s_t\right)\frac{u'\left(c^*\left(s^{t+1}\right)\right)}{u'\left(c^*\left(s^t\right)\right)} \text{ if } v_b\left(s^{t+1}\right)=0 \text{ and } v_l\left(s^{t+1}\right)\geq 0;$$

$$q^*\left(s^{t+1}|s^t\right) = \frac{1}{1+r}\pi\left(s_{t+1}|s_t\right) \text{ if } v_l\left(s^{t+1}\right)=0 \text{ and } v_b\left(s^{t+1}\right)>0.$$

Note that the price can be expressed alternatively as follows (see Appendix for details):

$$\begin{aligned} q^*\left(s^{t+1}|s^t\right) &= \max\left\{\beta\pi\left(s_{t+1}|s_t\right)\frac{u'\left(c^*\left(s^{t+1}\right)\right)}{u'\left(c^*\left(s^t\right)\right)},\left(\frac{1}{1+r}\right)\pi\left(s^{t+1}|s^t\right)\right\} \qquad (10) \\ &= \max\left\{\beta\pi\left(s_{t+1}|s_t\right)\frac{1+v_l(x_{t+1},s_{t+1})}{(1+v_b(x_{t+1},s_{t+1}))\eta},\left(\frac{1}{1+r}\right)\pi\left(s_{t+1}|s_t\right)\right\} \\ &= \left(\frac{1}{1+r}\right)\pi\left(s_{t+1}|s_t\right)\max\left\{\frac{1+v_l(x_{t+1},s_{t+1})}{1+v_b(x_{t+1},s_{t+1})},1\right\}. \end{aligned}$$

Since we impose borrowing limits that bind exactly when the participation constraints are binding in the optimal fund contract, $q\left(s^{t+1}|s^t\right) = q^*\left(s^{t+1}|s^t\right)$ satisfies the Euler conditions in the competitive equilibrium characterized above.

The prices $q\left(s^{t+1}|s^t\right)$ derived from the allocation of consumption and labor of the optimal fund contract define the ***price of a one-period bond***:

$$q^f\left(s^t\right) = \sum_{s^{t+1}|s^t} q\left(s^{t+1}|s^t\right)$$

This is the implicit price that we can use to compare with the bond price in the incomplete markets economy. Alternatively, we can use the implicit interest rate: $r^f\left(s^t\right) = 1/q^f\left(s^t\right) - 1$ or the ***spread***: $r^f\left(s^t\right) - r$.

Notice that, if the lender's participation constraint is not binding at s^{t+1}: $(1+v_b(x_{t+1}, s_{t+1}))^{-1} \leq 1$. Therefore, either there is *no spread* or *the spread is negative*. The latter occurs in the (2S) economy when lender's participation constraint is binding, in some s^{t+1}, as to make $\sum_{s^{t+1}|s^t} \pi\left(s_{t+1}|s_t\right)\left\{\frac{1+v_l(x_{t+1},s_{t+1})}{1+v_b(x_{t+1},s_{t+1})}\right\} > 1$; that is, when the market price of the lender's transfer, $\tau(s^t)$, is greater than the international market price at which he borrows and lends. In other words, *the spread* $r^f(s^t) - r < 0$ reflects the *the wedge* created by the lender's participation constraint; a wedge that aligns the market price with the lender unwillingness to lend. In particular, *when the spread is negative there is no borrowing* (i.e. $r^f\left(s^t\right) - r < 0 \Longrightarrow \tau\left(s^t\right) \geq 0$), since the riskless rate offers a better alternative to the lender.

Second, it is easy to see that given these asset prices and the normalized price of the consumption good not only support the optimal allocation of consumption for both agents $\{c^*\left(s^t\right), \tau^*\left(s^t\right)\}$, but also the labour allocation $\{n^*\left(s^t\right)\}$, while $\{e^*\left(s^t\right)\}$ satisfies the effort optimality condition as long as the borrower's value functions are the same in the *FSF* contract and in this competitive equilibrium, as we show below.

Third, we use the intertemporal budget constraints to construct the ***asset holdings*** that make the allocations in the optimal contract satisfy the present value budget, namely:

$$\begin{aligned}
a_b\left(s^t\right) &= \sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q^*\left(s^{t+n}|s^t\right)\left[c^*\left(s^{t+n}\right) - \left(\theta(s^{t+n})f\left(n^*(s^{t+n})\right) - G\left(s^{t+n}\right)\right)\right] \\
&= -\sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q^*\left(s^{t+n}|s^t\right)\tau^*\left(s^{t+n}\right) \\
a_l\left(s^t\right) &= \sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q^*\left(s^{t+n}|s^t\right)c_l\left(s^{t+n}\right) = \sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q^*\left(s^{t+n}|s^t\right)\tau^*\left(s^{t+n}\right) \\
a_l\left(s^t\right) &= -a_b\left(s^t\right).
\end{aligned}$$

In this economy, binding participation constraints provide us with the borrowing limits given by

(8) and (9). More precisely,

$$
\begin{aligned}
A_b\left(s^t\right) &= -\sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q^*\left(s^{t+n}|s^t\right)\left[\theta(s^{t+n})f\left(n_b^*(s^{t+n})\right)-G\left(s^{t+n}\right)\right] \\
&= -\sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q^*\left(s^{t+n}|s^t\right)\left(\tau^*\left(s^{t+n}\right)+c^*\left(s^{t+n}\right)\right) \\
A_l\left(s^t\right) &= Z \\
&= \sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t}\left(\frac{1}{1+r}\right)^t \tau^*\left(s^{t+n}\right)
\end{aligned}
$$

where the first equality refers to histories $\{s^{t+n}\}_{n=0}^{\infty}$ following a state s^t where the limited enforcement constraint of the borrower is binding (i.e. the borrower is indifferent between remaining in the *FSF* contract and autarky) and, similarly the last equality corresponds to histories following a state where the limited enforcement constraint of the lender, who values transfers at the risk-free interest rate, is binding.

The corresponding *recursive competitive equilibrium* for these *FSF* decentralized economies is also defined in the standard way as a set of policy functions: $c^*(a_b, s)$, $n^*(a_b, s)$, $e^*(a_b, s)$, $a_b'(a_b, s), \tau^*(a_l, s), a_l'(a_l, s)$ and value functions, W^{bf}, W^{lf}, that solve the agents problems for the corresponding Arrow security prices, $q\left(s'|s\right)$ and, finally, markets clear. In particular, the value functions $(W^{bf}(a_b, s)$, $W^{lf}(a_l, s))$ are the mirror image of the value functions $(V^{bf}(x,\, s), V^{lf}(x, s))$, since, given that $a_l = -a_b$, the dimension of the state (co-state) is the same and, as we have seen the allocations are the same; in fact, given that the both sets of value functions value the respective allocations the same, the effort decision must also be the same.

Finally, some *FSF* accounting is also useful. Paralleling the discussion of the *incomplete markets,* the ***primary surplus*** (primary deficit if negative) is given by

$$
\sum_{s'|s} q\left(s'|s\right) a_b'\left(a_b(s), s'\right) - a_b(s) = \tau(x, s).
$$

5 Calibration

5.1 Functional Forms, Shock Processes and Parameter Values

The model period is assumed to be one quarter. To make the different contracts comparable, we choose the same parameter values across economies whenever this is possible.

The utility of the borrower is additively separable in consumption and leisure. In particular, we assume

$$
\log\left(c\right) + \frac{\gamma\left(1-n\right)^{1-\sigma}}{1-\sigma} - \omega e^2
$$

The preference parameters are set to $\sigma = 0.8$ and $\gamma = 2.5$. These are used, together with the discount factor $\beta = 0.90$, to match the correlation between consumption, hours and GDP. The interest rate is set to $r = 0.035$, the average real bond yield of German 10 year maturity bonds. Note that this implies a different discount factor for the lender of $\frac{1}{1+r} = 0.9662$, as well as a growth rate for the relative Pareto weight of the borrower of $\eta = 0.9315$ in the optimal contract. Regarding the technology, we assume that $f(n) = n^{\alpha}$ with the labor share of the borrower set to $\alpha = 0.566$ to match the average labor share across the ' euro-stressed' countries. The $\zeta(e)$ function determining how effort increases the likelihood of good realizations of the G^c shock is $-\exp(-\rho e)$. The participation constraint of the lender in the *FSF* contract is set to $Z = 0$, a very tight level. Finally, the probability that the borrower comes back to the market upon default is set it to $\lambda^i = 0.12$ in the incomplete market model with default, while we assume that *FSF-exit is irreversible*, therefore we set $\lambda^f = 0$. Furthermore, in both models, the default penalty takes the form

$$\theta^p = \begin{cases} \bar{\theta}, & \theta_t \geq \bar{\theta} \\ \theta, & \theta < \bar{\theta} \end{cases} \quad \text{with } \bar{\theta} = \psi \mathbb{E}\theta,$$

where $\psi = 0.86$. The latter two parameters, together with the discount factor β are chosen to match jointly the PIGS average debt to GDP ratio, spread level and spread volatility. Finally, the parameters of the long term bond (δ, κ) are set to $\delta = 0.812$ and $\kappa = 0.025$ to match the average maturity and the average coupon rate (coupon payment to debt ratio) of long term debt. The following table summarizes all the parameter values.

Table 1: Parameter Values

α	β	σ	γ	r	λ^i	ψ	δ	κ	Z
0.566	0.9	0.80	2.5	0.035	0.12	0.86	0.812	0.025	0.0

The technology shock θ is assumed to be a Markov Switching process that was first estimated for world TFP by Bai and Zhang (2010). The latter authors first calculate time series of TFP for many different countries using Solow residuals. If $\{\log \theta_{it}\}$ denotes the logged TFP, this is then fitted to the following panel Markov regime switching (MRS) model

$$\log \theta_{it} = (1 - \rho(s_{it}))\mu(s_{it}) + \rho(s_{it}) \log \theta_{it} + \sigma(s_{it})\varepsilon_{it},$$

where $s_{it} \in \{1, \ldots, S\}$ denote the regime of country i at time t, $\mu(s_{it})$, $\rho(s_{it})$, and $\sigma(s_{it})$ are functions of the regime, and $\varepsilon_{it} \overset{\text{iid}}{\sim} N(0,1)$. The country specific regime s_{it} is independent in the cross-section, and follows a Markov chain over time, with an $S \times S$ regime transition matrix P.

Since our model does not have any capital accumulation, we first calculate time series for the labor productivity data for the 5 ' euro-stressed' countries (Portugal, Ireland, Italy, Greece and Spain). We then estimate the model above by adapting the expectations maximization (EM) algorithm outlined in Hamilton(1989) to our setup, combined with a more efficient procedure of Hamilton (1994) to calculate the smoothed probabilities of latent regimes. We set $S = 3$ for the panel MRS model in our estimation. Because the likelihood function of the model is highly nonlinear, the EM algorithm of likelihood maximization may be stuck at a local maximum. To overcome this potential deficiency, we randomize the initial point in the parameter space for 1,000 times. The estimated parameters of the Markov Switching Process are displayed below:

Table 2: Parameters of the Markov Switching Process for ' euro-stressed' LP

All Countries	$\mu(s)$	$v(s)$	$\rho(s)$	P	$s=1$	$s=2$	$s=3$
$s=1$	7.26	0.016	0.78	$s=1$	0.94	0.00	0.06
$s=2$	7.41	0.008	0.92	$s=2$	0.07	0.93	0.00
$s=3$	8.28	0.013	0.97	$s=3$	0.00	0.13	0.87

Finally, the process is then discretized into a 9 state Markov chain, with three values in each regime.

We consider a simple specification for the cyclical component G_c of the government consumption shock G. In particular, G_c has a state space of $\mathcal{G}_c = \{G_c(1), G_c(2), G_c(3)\}$, with $G_c(1) > G_c(2) > G_c(3)$, and the transition matrix for G_c is pinned down by two parameters[10]:

$$\pi^{G_c} = \begin{bmatrix} \phi & \frac{2}{3}(1-\phi) & \frac{1}{3}(1-\phi) \\ 2\eta & \phi & 1-\phi-2\eta \\ \eta & 1-\phi-\eta & \phi \end{bmatrix}$$

The parameters of the transition matrix are set to $\phi = 0.9$ and $\eta = 0.005$. These parameters, together with the state space for the shock, are used to match several moments of current government expenditures, such as the G to GDP ratio, the persistence of the observed government consumption, and the relative volatility of government consumption with respect to output. The resulting transition matrix and government shock values of G^c are given below:

$$\pi^{G^c} = \begin{bmatrix} 0.9 & 0.067 & 0.033 \\ 0.01 & 0.9 & 0.09 \\ 0.005 & 0.095 & 0.9 \end{bmatrix}$$

$$G^c = \begin{bmatrix} 0.03 & 0.013 & 0.01 \end{bmatrix}$$

For the iid component G_d to government expenditures, we simply assume that is uniformly distributed over $[-\bar{m}, \bar{m}] = [-0.001, 0.001]$. In particular, we discretize G_d into N_d equally spaced grid points $\{G_d(1), \ldots, G_d(N_d)\}$ over the previous interval, and set $\Pr(G_d(i)) = 1/N_d$ for all i. Using G_c and the discretized values of G_d, the discretized G shock can be constructed according to:

$$G_{(i-1)N_c+j} = G_d(i) + G_c(j), \quad i = 1, \ldots, N_d, j = 1, \ldots, N_c,$$

where $N_c = 3$. Moreover, with some slight abuse of notation, we define the transition matrix π^G of the discretized G shock to be the Kronecker product of two matrices:

$$\pi^G = \pi^{G_c} \otimes \pi^{G_d},$$

where π^{G_c} is simply an $N_d \times N_d$ matrix with all entries equal to $1/N_d$. Defining π^G this way follows directly from the fact that G_c is independent of G_d. As noted by Chatterjee and Eyigungor (2012), the iid component improve considerably the convergence properties of the model with incomplete markets and default. We also use it to match, together with G^c, the relative volatility of government expenditures.

As stated earlier, we assume that effort only affects the Markovian government liabilities G^c. Given the current G^c there are two possible distributions of tomorrow's $\pi^b(\cdot|G^c)$ and $\pi^g(\cdot|G^c)$, and $\pi^g(\cdot|G^c)$ first order stochastically dominates $\pi^b(\cdot|G^c)$ for all G, with $\pi^G(G'|s, e) = \pi^G(G'|G^c, e)$ and

$$\pi^G(G'|G^c, e) = \exp(-\rho e))\pi^b(G'|G^c) + (1 - \exp(-\rho e))\pi^g(G'|G^c)$$

[10]See Appendix for further details.

Recall that that government expenditure is ordered in a decreasing way. This implies that increasing effort, ceteris paribus, increases the probability of low government expenditure. Note that this functional form implies simple expressions for $\frac{\partial \pi^G(G'|G,e)}{\partial e}$ and $\frac{\partial^2 \pi^G(G'|G,e)}{\partial e \partial e}$ as follows:

$$\frac{\partial \pi^G(G'|G,e)}{\partial e} = \rho \exp(-\rho e) \left[\pi^g(G'|G) - \pi^b(G'|G)\right]$$

and

$$\frac{\partial^2 \pi^G(G'|G,e)}{\partial e \partial e} = -\rho^2 \exp(-\rho e) \left[\pi^g(G'|G) - \pi^b(G'|G)\right]$$

5.2 Data Sources and Measurement

The primary data source we use is the AMECO dataset. We use annual data for the 5 ‘ euro-stressed’ countries, and except for a few series, the sample coverage is 1980–2015. Table 2 provides a summary of the data sources and definitions. We construct model consistent measures based on the raw data. In what follows, we detail on the sources and measurement methods.

5.2.1 National accounts variables

For the aggregate output Y_{it}, private consumption C_{it} and government consumption expenditure G_{it} of each country, we use directly the corresponding data series from AMECO over 1980–2015, measured in constant prices of 2010 euros. For the aggregate labor input n_{it}, we use two series from AMECO, the aggregate working hours H_{it} and the total employment E_{it} of each country over the period 1980–2015. The only exception is Greece, since the values are missing in AMECO between 1980–1982. To supplement the greek data, we use working hours data from the Penn World Table 8.1. We calculate the normalized labor input according to $n_{it} = H_{it}/(E_{it} \times 5200)$, assuming 100 hours of allocatable time per worker per week. However, for most parts of the data moments computations, we use H_{it} directly, since the per worker annual working hours do not show a significant cyclical pattern and the trend and level do not affect the computation of the moments.

5.2.2 Government bond variables

We use the end-of-year government debt to GDP ratios in AMECO to measure the indebtedness of the ‘ euro-stressed’ countries. The government debt is defined as the general government consolidated gross debt. This is conceptually different from the debt in the model, which corresponds to national debt more closely. nevertheless, we use the gross debt measure, as it provides a consistent measure across countries and is arguably an upper limit on the indebtedness of the government. We use the nominal long-term bond yields in AMECO to measure the nominal borrowing costs of the ‘ euro-stressed’ countries. To obtain the real bond yields, we subtract the realized yearly inflation rate from the nominal yield, thus assuming perfect foresight of agents in each country. We use the GDP delator to measure inflation rate. We calculate the real yields for Germany in the same way, and take the difference between the real yields of the ‘ euro-stressed’ countries and those of Germany as the measure of bond spread. The information on the maturity structure of the government debt for the ‘ euro-stressed’ countries is quite limited. We are able to find average years to maturity for the five countries only from OECD government debt database. The measurement is for the total outstanding central government debt. The time coverage is unequal across countries: 1998–2010 for Ireland and Greece, 1996–2010 for Portugal, 1991–2010 for Spain, and 1990–2010 for Italy.

5.2.3 Fiscal variables

To measure the government's fiscal performance, we use directly the series of primary surplus to GDP ratio in AMECO for the ' euro-stressed' countries. The online dataset of AMECO only contains primary surplus series from 1995 onwards, and for 1980–1994, we use values reported in European Commission General Government Data (GGD) 2002. There are some differences in the statistical definitions for the primary surplus series. In particular, the AMECO series is based mainly on the ESA 2010 standards, supplemented by ESA 1995 for a few observations between 1995 to 2000, while the GGD series is based on the former definition. However, for overlapping years reported in GGD 2002, the average difference of the two definitions is limited. For the model, the theoretically consistent measure of the primary surplus is simply $y - c - G$, i.e., the total saving of the economy. Recall that the primary surplus is defined as government surplus minus interest payments. Alternatively, by the government's budget constraint, the primary surplus can be expressed as the net lending by the government, i.e., the difference between revenue of newly issued debt and payments on interests and retiring debt. For the economy with incomplete markets and default we are considering, this equals to $q_t(b_{t-1} - b_t) - (1 - \delta + \delta\kappa)b_t$, and by the economy's budget constraint, the last expression is just equal to $y_t - c_t - G_t$, which is the measure we use.

5.2.4 Labor share

We use various data series from AMECO to compute the labor share of annual output for each of the ' euro-stressed' over the period 1980–2015. First, we use nominal compensation to employees of the total economy in AMECO (labeled by UWCD) to measure the labor income for employees. Second, to measure the labor income for self-employed people, we take the difference between two AMECO series, UOGD and UQGD, where the former is gross operating surplus and the latter is the same measure net off imputed compensation for the self-employed population. We define the total labor income as the sum of the labor income for employees and self-employed, i.e., UWCD + UOGD-UQGD. Finally, the labor share is calculated as the ratio of labor income to nominal GDP.

5.2.5 Labor productivity

Given the production function, $y = \theta n^{\alpha}$, we measure the labor productivity of country i at time t according to $\theta_{it} = Y_{it}/H_{it}^{\alpha}$, or equivalently, $\log\theta_{it} = \log y_{it} - \alpha\log n_{it}$. Note that we use a common α for all ' euro-stressed' countries. Let $\hat{\theta}_{it}^{o}$, or $\log\hat{\theta}_{it}^{o}$, denote this measure of the original level for labor productivity. To compute the data moments involving the labor productivity, we use the HP-filter to detrend the sample productivity $\{\log\hat{\theta}_{it}^{o}\}$. Moreover, as described earlier, we use a Markov regime switching model to estimate the productivity process. Before taking the data to the model, we adjust the original sample in the following two steps:

1. We take out a country specific linear time trend in the $\{\log\hat{\theta}_{it}^{o}\}$ series. This is a standard practice in the estimation of a productivity process.

2. After detrending, we further add or subtract a constant to $\{\log\hat{\theta}_{it}^{o}\}$ for each i so that the resulting series has the same sample average over i. This is to prevent the level differences in $\{\log\hat{\theta}_{it}^{o}\}$ across i from producing spurious regime switching behavior in the estimation procedure, so as

to ensure the regime switching effect in the estimated productivity process is caused by the time series variation in the observed productivity series for each country.

We denote the adjusted sample productivity by $\{\log \hat{\theta}_{it}\}$, which is then used in the estimation of the MRS model discussed earlier.

6 Numerical Results

This section discusses the numerical results without moral hazard (i.e. $v(e) = 0$). We compare the incomplete markets economy with default (**IMD**) and the economy with a *FSF* with two sided limited of commitment (**2S**). We first present calibration results in Table 1 and the *policy functions* for both economies in Figures (1) - (4). To better understand how these economies work, we show representative paths of both economies, subject to the same sequence of shocks in steady state, in Figures (5) - (6).Finally, we study how both economies respond to a combined negative shock when they are in steady state: Figures (7) - (8)[11]. TFP shocks are labeled $e_i, i = 1, \ldots, 9$ where $e_i < e_{i+1}$ and G shocks are labeled $g_j, ij = 1, \ldots, 9$ where $j_i > e_{j+1}$; that is (e_1, g_1) is the worst combination of shocks and, increasingly, (e_9, g_3) is the best combination of shocks.

6.1 Calibration results

The following Table 1 provides an exhaustive account of our calibration results.

6.2 Policy Functions

The core of the analysis is given by the study of the different optimal policy functions. Figures (1) - 2) displays the policy functions for the main variables for the incomplete markets economy with default (**IMD**), and Figures (3) - 4) the economy with a *FSF* and two sided lack of commitment (**2S**). The policies are plotted for selected values of shocks ($s = (\theta, G) = (e, g)$) in: (**IMD**) as function of the level of debt, and in (**2S**) as a function of the relative Pareto weight of the borrower, which we denote *Pareto weight* in what follows.

As it can be seen in Figure (1), in the (**IMD**) economy consumption and debt choices, as well as borrower's value are monotone with respect to shocks at all levels of debt, while the labor choice losses its monotonicity at high levels of debt (high negative values), showing that when a country is heavily indebted works harder when it is subject to negative productivity shocks. The economy with default (**IMD**) distorts the previous policies by the choice of default, which is shown as a discontinuity in $b^{i\prime}(s, b)$ and a corresponding constant autarkic choice for bad shocks. It should be noticed that the labor choice reversals occur at levels of debt just below the default level and the 'working harder in bad times' persists in autarky. Figure (2) shows the bond prices for different shocks.

Figures (3) - (4) show the policies of the fund contract with two-sided limited commitment. It is interesting to compare the evolution of relative Pareto weights in the upper-left panel of (3) and the consumption policies in (4). With limited enforcement, the relative Pareto weight of the borrower cannot decrease indefinitely in (3) but even if in principle one could assign a relatively hight weight to the borrower, limited enforcement or commitment of the lender sets upper bounds on how much Pareto weight can be given to the borrower. The figure also shows how these bounds depend on the

[11] Figures (9) - (13) in the Appendix show how the economies respond to one of the two shocks.

Table 1: Benchmark calibration

Moments	Data	Model (IMD)
Mean		
Debt to GDP ratio	77.39%	66.7%
Real bond spread	0.41%	1.59%
Real risk free rate	3.53%	3.53%
Maturity	5.32	5.32
G to GDP ratio	20.26%	17.66%
—range of G/GDP	[10.19%, 45.8%]	[12.84%, 34.37%]
Primary surplus to GDP ratio	−0.85%	1.7%
Fraction of working hours	19.23%	22.94%
Share of labor income	0.566	0.566
Volatility		
$\sigma(C)/\sigma(Y)$	1.12	1.65
$\sigma(N)/\sigma(Y)$	0.92	1.20
$\sigma(G)/\sigma(Y)$	1.03	3.29
$\sigma(PS)/\sigma(Y)$	1.20	1.08
σ(real spread)	1.65%	0.54%
Correlation		
$\rho(C, Y)$	0.80	0.42
$\rho(N, Y)$	0.72	0.16
$\rho(PS/Y, Y)$	0.23	0.07
$\rho(G, Y)$	0.36	0.34
$\rho(\text{real spread}, Y)$	-0.16	-0.27
$\rho(G, \theta)$	0.16	0.00
$\rho(G_t, G_{t-1})$	0.94	0.75

shocks and how relative borrower's weights (and consumptions) decrease when limited enforcement constraints are not binding. In particular, the worst shock, $e_9 g_3$, defines a lower bound on the weight (and consumption) of the borrower.

6.3 Computational experiments.

In this section, we discuss the simulation results to compare both economies. The first, denoted *Business Cycle Paths* (5) - (6) , is a long-run simulation. In the second, denoted *Impulse Responses* (7)- (8), we assume that the economy is hit by negative (θ, G) shocks (e_1, g_1) but then all shocks after the initial period follow a realization of the (θ, G) stochastic process; therefore we report the average impulse response from 500 simulations. The initial endogenous conditions are randomly chosen from the stationary distribution.

More specifically, Figure (6) shows the asset positions and spreads of a typical realization of the

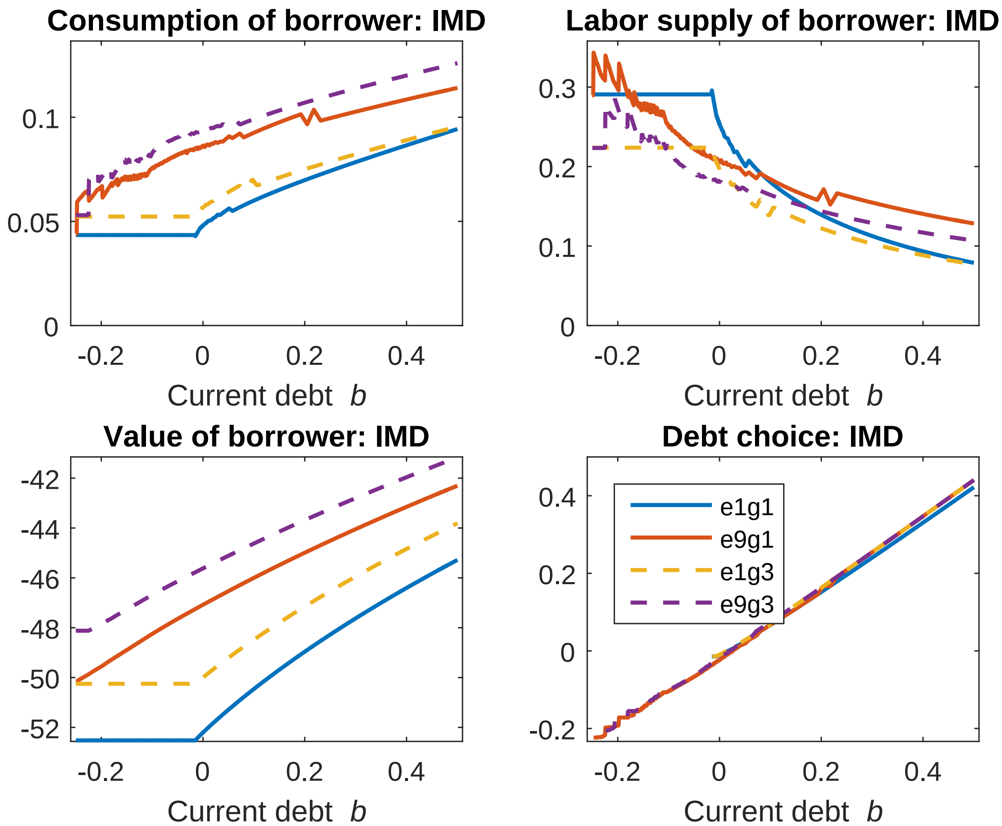

Figure 1: IMD policy functions: allocations.

incomplete markets economy where default takes place and before that we observe a sequence of episodes with *positive spreads*. However, we do see few episodes of *negative spreads* corresponding to periods with relatively high negative primary surpluses in which the lender's participation constraint is binding.

Finally, Figures (7)- (8) show how the two economies react to a transitory combination of bad (θ, G) shocks. The crisis is more severe in the incomplete markets economy with default (**IMD**) than in the economy with a *FSF* and two-sided limited commitment.

6.4 Welfare comparisons

To complete the previous analysis we now provide a quantitative welfare comparison between the economy with a *FSF* and two-sided limited commitment (**2S**) and the incomplete markets economy with default (**IMD**); in other words, a measure of the value of substituting sovereign debt financing by sovereign financing through a well designed *FSF*.

We compute a simple measure, χ, of *consumption equivalence*, taking advantage of the decomposition of the welfare functions $V_b^j = V_{b,c}^j + V_{b,n}^j$, where $j = i, f$, corresponds to the incomplete markets (**IMD**) and the *FSF* (**2S**) economies, respectively and the subscripts c and n correspond to

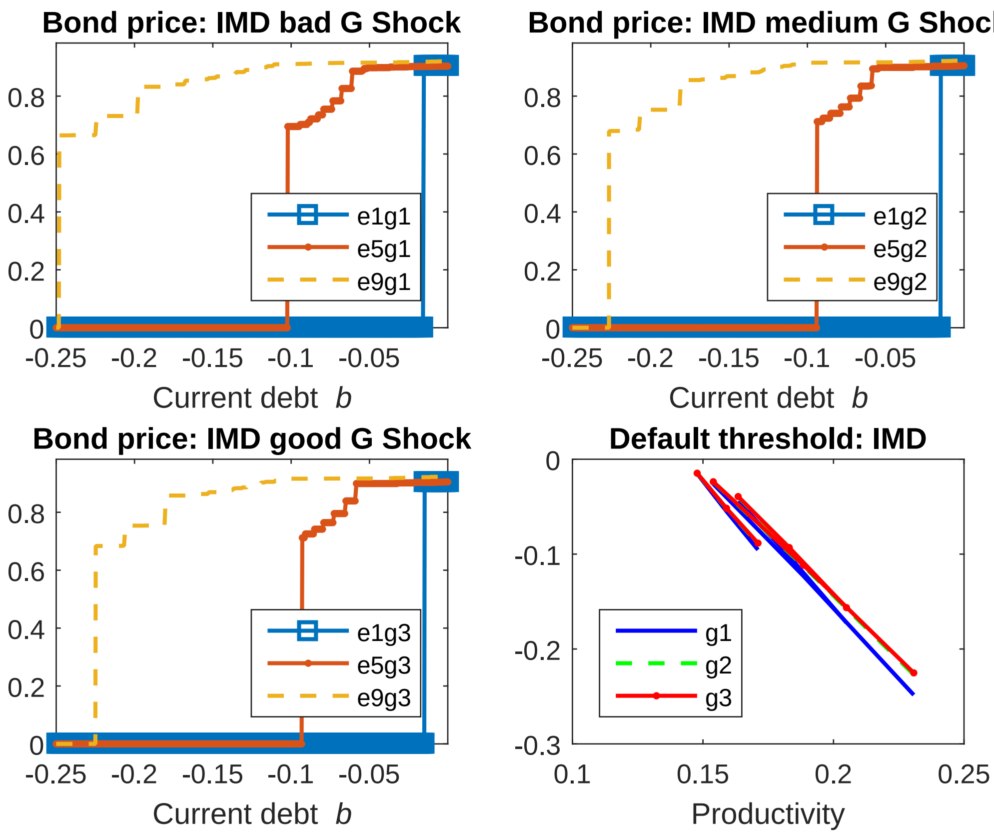

Figure 2: IMD policy functions: bond prices.

the corresponding decomposition between consumption and labour effort. In particular,

$$V^j_{b,c} = \log\left(c^j\right) + \beta E V^{j\prime}_{b,c} = E_0 \sum_{t=0}^{\infty} \beta^t \log(c^j_t)$$

$$V^j_{b,n} = \gamma \frac{\left(1-n^j\right)^{1-\sigma}}{1-\sigma} + \beta E V^{j\prime}_{b,n}$$

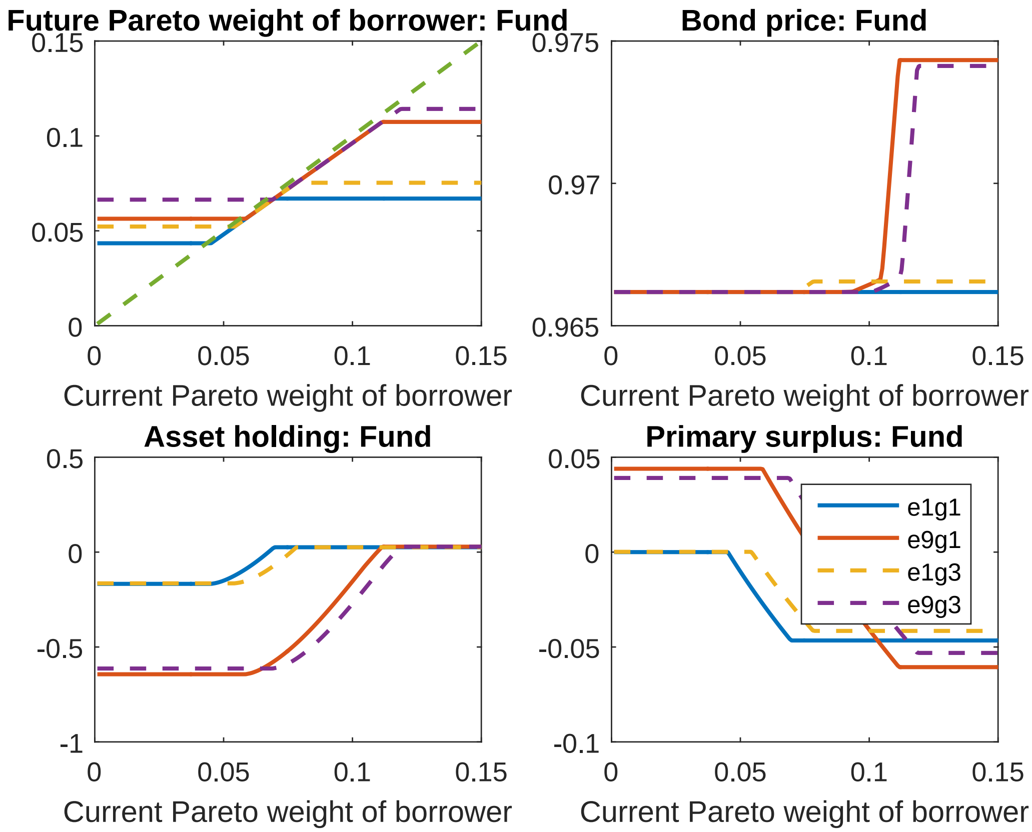

Figure 3: *FSF* policy functions: Pareto weights and assets.

The χ measure solves the following *consumption equivalence*:

$$
\begin{aligned}
V_b^f &= E_0 \sum_{t=0}^{\infty} \beta^t \log((1+\chi)c_t^i) + V_{b,n}^i = \\
&= \frac{\log(1+\chi)}{1-\beta} + E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t^i) + V_{b,n}^i = \\
&= \frac{\log(1+\chi)}{1-\beta} + V_{b,c}^i + V_{b,n}^i \\
&= \frac{\log(1+\chi))}{1-\beta} + V_b^i \\
&\Rightarrow (1+\chi) = \exp\left(\left(V_b^f - V_b^i\right)(1-\beta)\right).
\end{aligned}
$$

In other words, χ compensates in consumption the differences in consumption and labour across the two economies.The following Table reports the values of χ, computed from directly from the policy functions, when initial assets are zero.

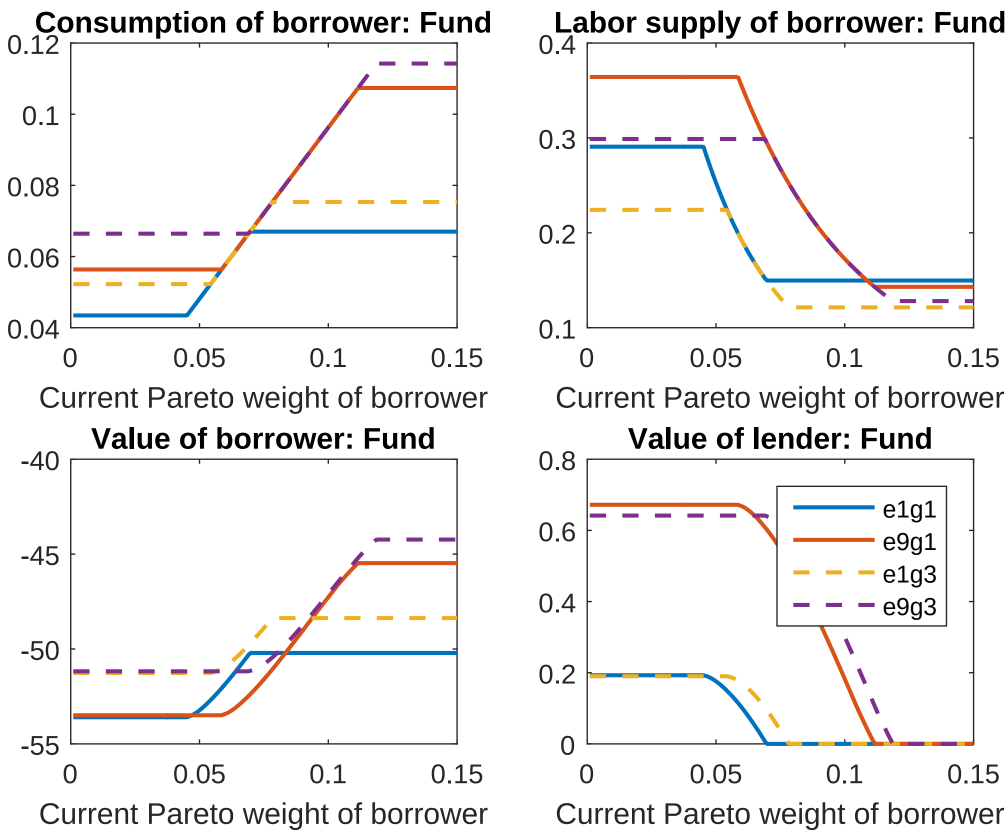

Figure 4: *FSF* policy functions: allocations and values.

Shocks (θ, G)	Welfare Gain	b^{max}: Markets	b^{max}: Fund
$(\theta_l, G_h) = (0.148, 0.03)$	11.9	20.5	227.8
$(\theta_m, G_h) = (0.183, 0.03)$	10.27	107.3	344.9
$(\theta_h, G_h) = (0.231, 0.03)$	9.8	196.5	494.0
$(\theta_l, G_l) = (0.148, 0.01)$	9.4	23.11	259.7
$(\theta_m, G_l) = (0.183, 0.01)$	8.3	113.7	376.4
$(\theta_h, G_l) = (0.231, 0.01)$	8.3	200.3	526.5

Notice that the welfare gains are huge, which is consistent with very high levels of sustainable debt, within the *Financial Stability Fund*, even if there is a *tight limited enforcement constraint* for the lender (i.e. $Z = 0$). However, these high numbers are due to three key elements: i) the relative impatience of the borrower; ii) the existence of a 'default penalty', which is the same than in the IMD economy, $\theta^P \neq \theta$ and iii) the no return assumption: $\lambda^f = 0$. The last two elements make the economy with a *FSF* to be very close to an economy with *full commitment* and with *full commitment*, when the borrower is more impatient than the lender, the level of debt is very high, since it is asymptotically paid as consumption of the borrower decays. Obvioulsy, changes in these parameters – in particular, eliminating the 'default penalty' for the *FSF* $\theta^P \neq \theta$ – changes the result quantitatively. The change

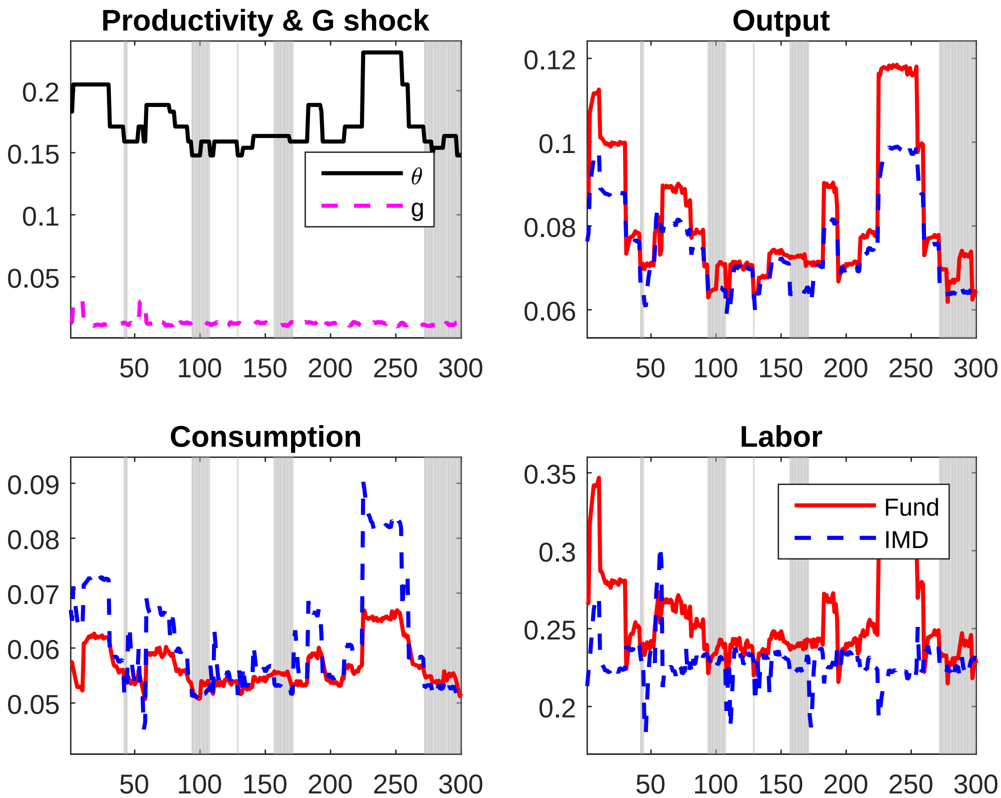

Figure 5: IMD vs. FSF Business Cycle Paths: shocks and allocations,

can also be qualitative – that is, the IMD economy can be relatively more efficient than the *FSF* – if a *tight* Z is associated with a low default penalty $\theta^P \approx \theta$ and it is possible to restart a *FSF contract*, $\lambda^f > 0$.

7 Conclusions

By developing and computing a model of a *Financial Stability Fund* we have provided a useful instrument to study the gains of implementing it, as well as how it should be implemented; to estimate how different sovereign debt crisis could be, and can be handled, with it. As usual, practical implementation has complexities beyond our analysis, but if anything this only underlines the need to fill the gap between the ample experience with debt financing and fund interventions and the almost inexistent theory. Part of this gap can be explained by the need to use advanced tools of dynamic contract design to properly model a *FSF*. To bring these tools to develop such model, to contrast it with standard sovereign debt financing, and to calibrate the model to the experience of the Eurozone ‘ stressed’ countries, we think is the main contribution of this paper. More work needs to be done, in particular in making our *FSF* model a workable proposal – or some variant of it – within the current EU and Eurozone legal and political framework. Nevertheless, no matter how refine and feasible the

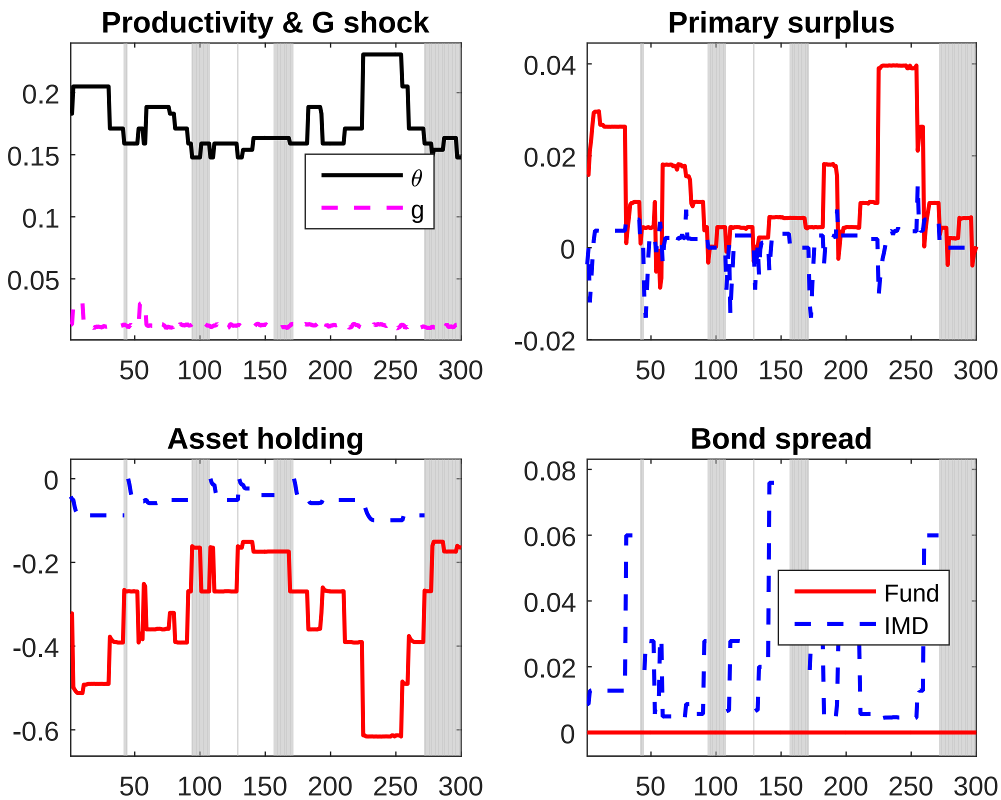

Figure 6: IMD vs. FSF Business Cycle Paths: shocks and assets.

proposal becomes, at the end political will be needed for its implementation.

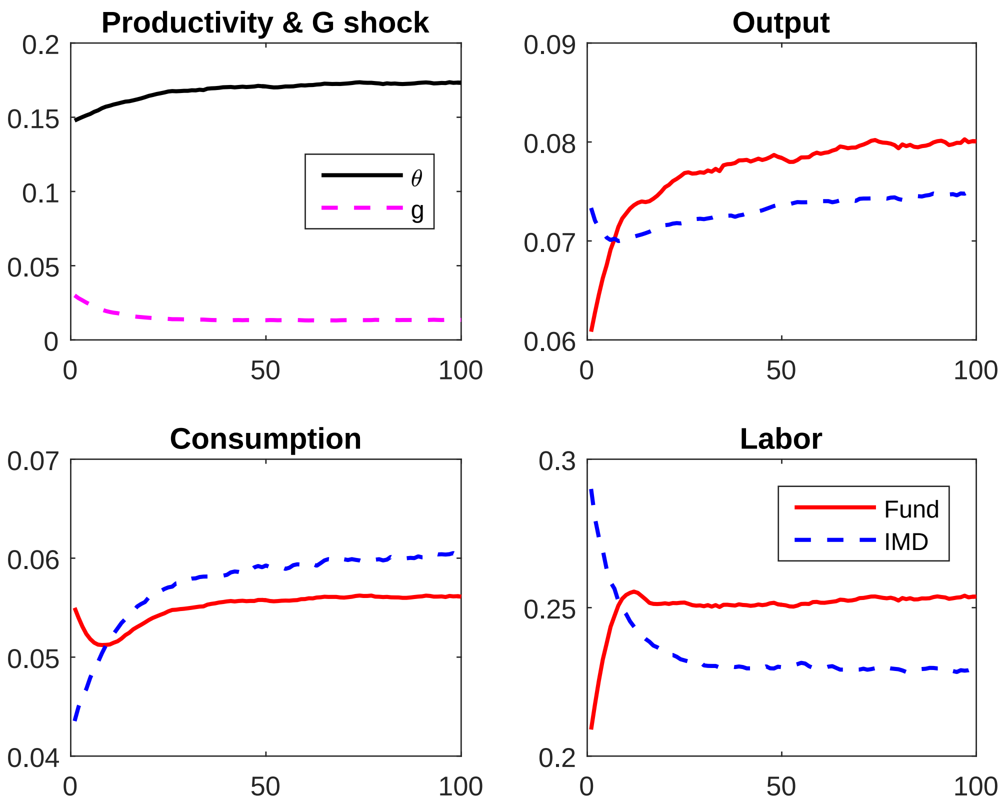

Figure 7: IMD vs. FSF: combined shock impulse-responses: allocations.

8 Appendix

8.1 Data sources

8.2 Additional experiments

8.3 Solution Method

8.3.1 The Solution of the IMD

In what follows, we describe the computational algorithm to solve for the IMD model with no moral hazard.

Solving for the labor supply For given (s, b) and b', we can solve for the optimal labor from the optimality condition. If the borrower chooses not to default, the optimal labor supply n^* solves:

$$h(n) \equiv (\theta n^{\alpha} - \chi)\, n^{1-\alpha} - \vartheta(1-n)^{\sigma} = 0$$

where $\vartheta = (\theta\alpha)/\gamma > 0$ and $\chi = G - (1 - \delta + \delta\kappa)b + q(s, b')(b' - \delta b)$. Since $h(1) = (\theta - \chi)$ and $h(0) = -\vartheta < 0$, there exists an $n^* \in (0, 1)$ such that $h(n^*) = 0$ and $c^* > 0$ if and only if $\theta - \chi > 0$. It

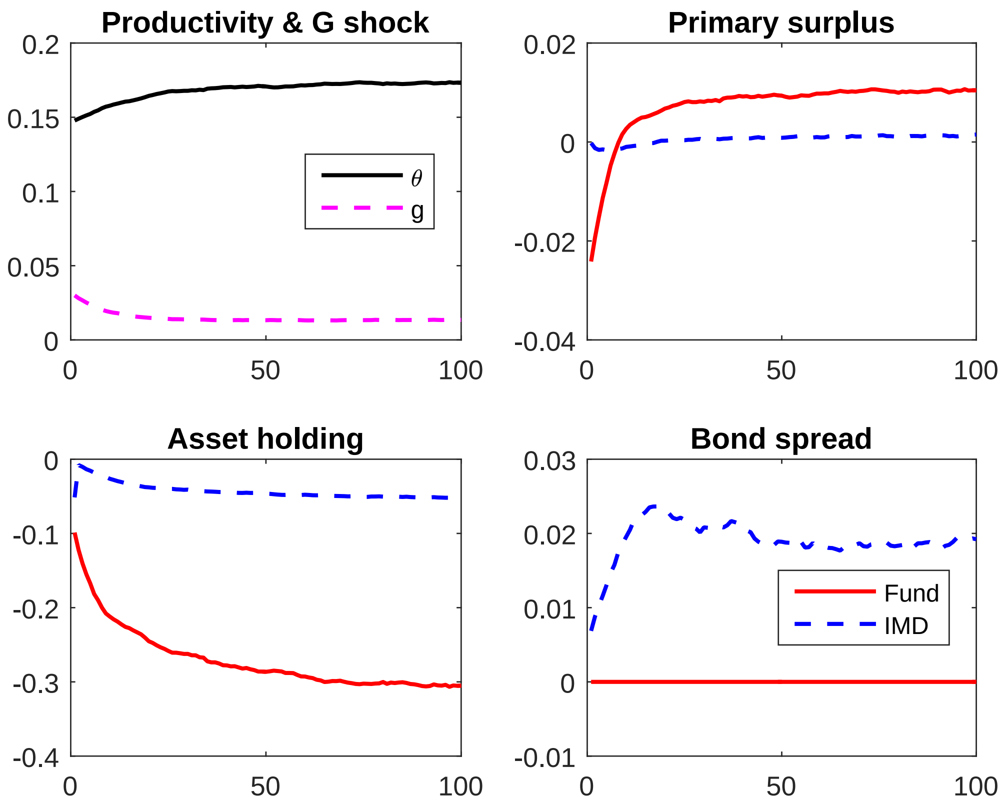

Figure 8: IMD vs. FSF: combined shock impulse-responses: assets.

is easy to show that n^* is unique. If the borrower chooses to default, we can use the same condition with $\vartheta = \theta^p \alpha/\gamma$ and $\chi = G$.

In what follows, we denote by $N_{\mathrm{nd}}(s, b, b')$ the optimal labor supply in the case of no default, given the current state (s, b) and the bond choice for the next period b'; and we use $N_{\mathrm{d}}(s)$ to denote the optimal labor supply in the case of default. Here we have chosen to suppress the dependence of N_{nd} on the bond price $q(s, b')$ for two reasons: first, given any pricing function $q(\cdot)$, the specific value of the bond price is determined by (s, b'); and second, to enhance computational efficiency, we will rewrite $N_{\mathrm{nd}}(\cdot)$ as a function of θ and χ, where χ summarizes all the dependence of N_{nd} on G, b, b', and $q(s, b')$.

Solving the Bellman Equation To find a solution to the model, we combine equations (1)-(3) as well as the pricing equation in (4) into one Bellman equation of four functions: three value functions and one pricing function. We can then use backward induction to solve the functional equation. More precisely, let $V^{bi}(s, b; k-1)$, $V_n^{bi}(s, b; k-1)$, $V^{ai}(s; k-1)$, and $q(s, b'; k-1)$ denote the value and

Table 2: Data sources and definitions

Series	Time periods	Sources	Unit
Output	1980–2015	AMECO (OVGD)[a]	1 billion 2010 constant euro
Private consumption	1980–2015	AMECO (OCPH)	1 billion 2010 constant euro
Government consump.	1980–2015	AMECO (OCTG)	1 billion 2010 constant euro
Working hours	1980–2015	AMECO (NLHT)[b]	1 million hours
Employment	1980–2015	AMECO (NETD)	1000 persons
Government debt	1980–2015	AMECO EDP[c]	end-of-year percentage of GDP
Primary surplus	1980–2015	AMECO (UBLGIE)[d]	end-of-year percentage of GDP
Bond yields	1980–2015	AMECO (ILN)[e]	percentage, nominal
Inflation rate	1980–2015	AMECO (PVGD)	percentage, GDP deflator
Debt maturity	1990–2010	OECD[f]	years
Labor share	1980–2015	AMECO[†]	percentage

a Strings in parentheses indicate AMECO labels of data series.

b PWT 8.1 values for Greece in 1980–1982.

c General government consolidated gross debt; ESA 2010 and former definition, linked series.

d AMECO linked series for 1995–2015; European Commission *General Government Data* (GDD 2002) for 1980–1995.

e A few missing values for Greece and Portugal replaced by Eurostat long-term government bond yields.

f Differing time coverage across countries; see the text for details.

† Calculated based on various series on labor compensation; see the text for details.

pricing functions obtained in the k'th iteration. We first solve

$$V_n^{bi}(s,b;k) = \max_{c,b'} \; U(c, 1-N_{\text{nd}}(s,b,b';k)) + \beta\mathbb{E}\left[V^{bi}(s',b';k-1)|s\right] \tag{1}$$
$$\text{s.t. } c + q(s,b';k)(b'-\delta b) \leq \theta[N_{\text{nd}}(s,b,b';k)^{\alpha} - G_c - G_d + (1-\delta+\delta\kappa)b,$$

and

$$V^{ai}(s;k) = U(c, 1-N_{\text{d}}(s)) + \beta\mathbb{E}\left[(1-\lambda)V^{ai}(s';k-1) + \lambda V^{bi}(s',0;k-1)|s\right] \tag{2}$$
$$\text{s.t. } c = \theta^p[N_{\text{d}}(s)]^{\alpha} - G_c - G_d,$$

so that

$$V^{bi}(s,b;k) = \max\{V_n^{bi}(s,b;k), V^{ai}(s;k)\}. \tag{11}$$

As explained earlier, we denote the labor supply function in the no default case by $N_{\text{nd}}(s,b,b';k)$ to make explicit the dependence of $N_{\text{nd}}(\cdot)$ on the bond pricing function $q(\cdot;k)$ in each iteration. This is a standard dynamic programming problem that delivers value and policy functions for consumption, labor and bond choices, as well as default decisions. Once we have these, we can update the pricing function via

$$q(s,b';k+1) = \mathbb{E}\left[(1-D(s',b';k))\frac{(1-\delta)+\delta[\kappa+q(s',b(s',b';k);k)]}{1+r}|s\right], \tag{12}$$

where $D(s,b;k)$ and $b(s,b;k)$ are the default and bond holding decisions obtained in iteration k. In general, this shows that $q(\cdot;k)$ is obtained in iteration $k-1$.

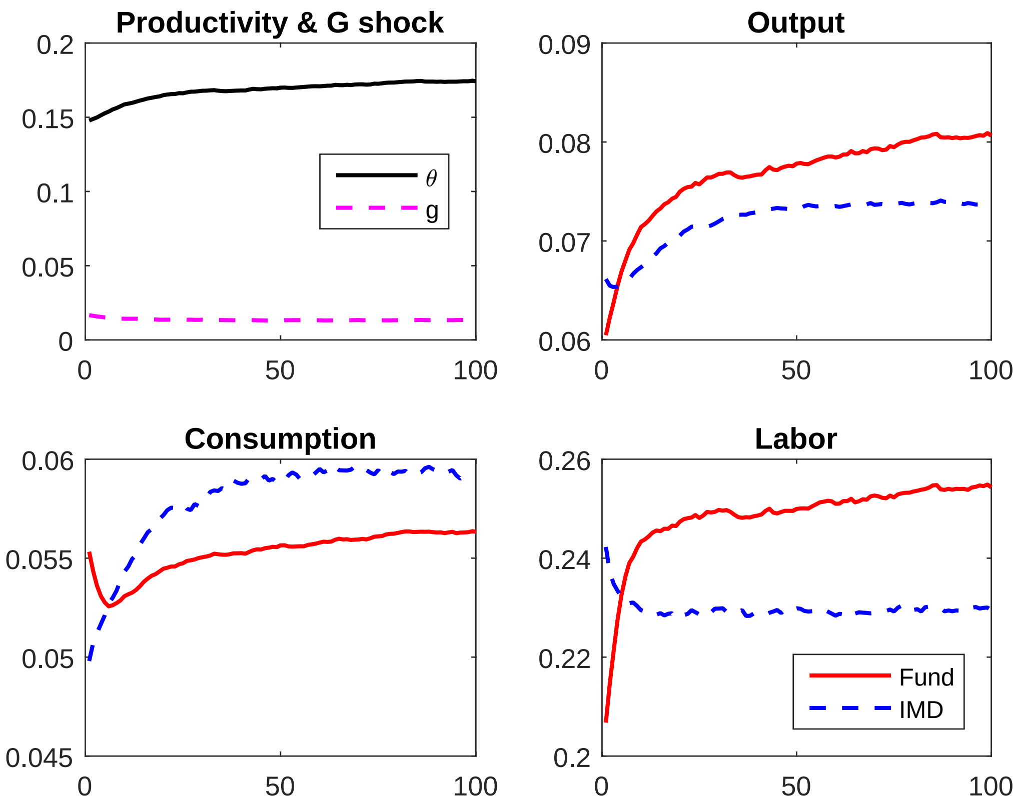

Figure 9: IMD vs. FSF: technology shock impulse-responses: allocations.

To implement the backward induction algorithm, we use discrete space value function iteration. Since (θ, G_c) is discrete by assumption, we only need to discretize G_d and b. In particular, we set G_d to be equally spaced over $[-\bar{m}, \bar{m}]$ with N_d grid points, and with equal probability on each grid point for simplicity. Moreover, we discretize the bond holding space $\mathcal{B}$ with N_b grid points. We iterate on the value function and the pricing function on the discretized space $\Theta \times \mathcal{G}_c \times \mathcal{G}_d \times \mathcal{B}$ until convergece, namely, until

$$\max |V^{bi}(s, b; k) - V^{bi}(s, b; k+1)| \text{ and } \max |q(s, b'; k) - q(s, b'; k+1)|$$

are both smaller than some convergence criterion. Moreover, we use two parameters $\zeta_V, \zeta_q \in [0, 1]$ to control the updating speed of $V^{bi}(\cdot)$ and $q(\cdot)$ as follows:

$$V^{bi}(s, b; k+1) = \zeta_V V^{bi}(s, b; k) + (1 - \zeta_V)\text{RHS of (11)},$$
$$q(s, b'; k+1) = \zeta_q q(s, b'; k) + (1 - \zeta_q)\text{RHS of (12)}.$$

Setting $\zeta_q > 0$ is useful for the convergence of $q(\cdot)$ as well.

Note that it is important to have a continuously distributed G_d to smooth off discrete changes in $D(s, b)$ and enhance the convergence properties of the model. In principle, we could keep G_d

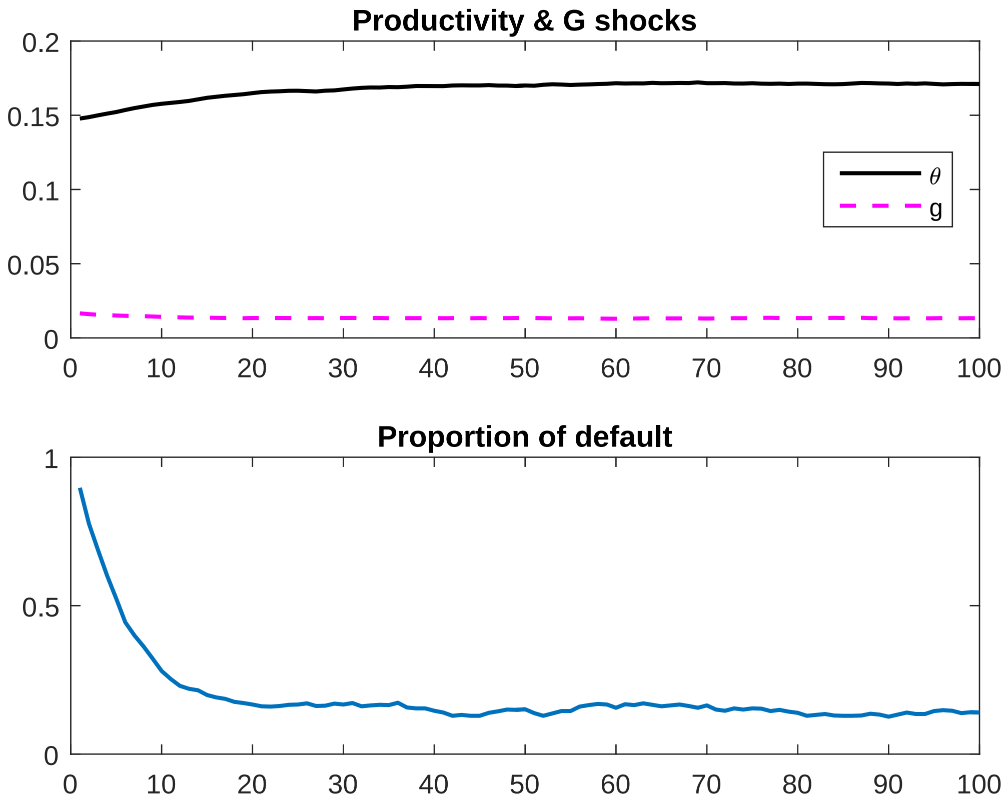

Figure 10: IMD: technology shock impulse-responses: defaults.

as a continuous state variable in the computation, and use the involved procedure of Chatterjee and Eyigungor (2012) to obtain the functions $D(\cdot, G_d, \cdot)$ and $b(\cdot, G_d, \cdot)$ accurately. Instead, we use a discrete approximation of G_d, which is straightforward to implement, and we find that such an approximation works good enough to improve the convergence properties of the algorithm to compute our model.

Note also that $q(s, b')$ does not depend on G_d, and this simplifies the iterations of $q(s, b'; k)$. Also, in the backward iteration, we use the fact that $q(s', b(s', b'; k); k)$ is simply the equilibrium bond price in state (s', b'), the value of which has already being computed in solving for the optimal bond choice $b(s', b'; k)$. Let $q^{\mathrm{e}}(s, b; k)$ denote the equilibrium bond price in state (s, b) under optimal bond choice $b(s, b; k)$, then (12) can be simplified into

$$q(s, b'; k+1) = \mathbb{E}\left[(1 - D(s', b'; k))\frac{(1-\delta) + \delta[\kappa + q^{\mathrm{e}}(s', b'; k)]}{1+r}|s\right].$$

The above expression implies that two equivalent ways of updating the bond prices. The first is to compute $q(\cdot; k+1)$ in the k'th iteration, after obtaining the default decision $D(\cdot; k)$. The second is to compute $q(\cdot; k+1)$ in iteration $k+1$ for each bond choice b', using default decisions obtained in the previous iteration. The latter will be useful when we implement the moral hazard case.

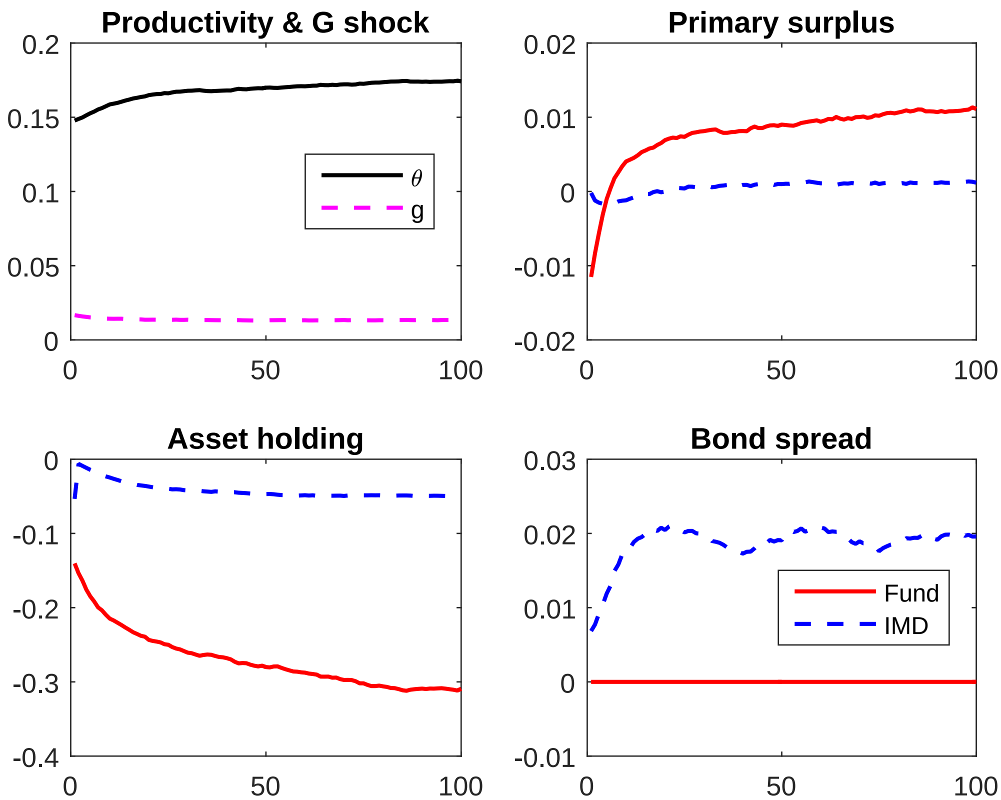

Figure 11: IMD vs. FSF: technology shock impulse-responses: assets.

Improving on Efficiency In the preceeding algorithm, we do the computation of optimal labor supply $N_{\mathrm{nd}}(s, b, b')$ under no default *within* the main loop. To improve on efficiency we can use an approximation of $N_{\mathrm{nd}}(s, b, b'; k)$. As shown before, for given (s, b, b') and bond pricing function $q(s, b'; k)$, the optimal labor n^* can be written as a function of θ and χ. Since $G > 0$, $0 \leq q(s, b') \leq \bar{q} = \frac{1-\delta+\delta\kappa}{1-\delta+r}$ and $b_{\min} < 0 \leq b_{\max}$, we have $\chi_{\min} \leq \chi \leq \chi_{\max}$, where

$$\chi_{\min} = G_{\min} + \bar{q}[b_{\min} - (1+r)b_{\max}],$$
$$\chi_{\max} = G_{\max} + \bar{q}[b_{\max} - (1+r)b_{\min}].$$

Therefore, we can discretize the interval $[\chi_{\min}, \chi_{\max}]$ into a fine grid $\mathcal{X}$ with N_χ equally spaced points, and then solve for n^* over the grid $\Theta \times \mathcal{X}$ once and for all *outside* the main loop. Denote this solution by $N^*_{\mathrm{nd}}(\theta, \chi)$. To evaluate $N_{\mathrm{nd}}(s, b, b'; k)$ within the loop, we can simply interpolate N^*_{nd} for the level of χ implied by (s, b, b').

8.3.2 The Solution of the *FSF*

Using the functional forms above, the equilibrium conditions for the *FSF* can be rewritten as:

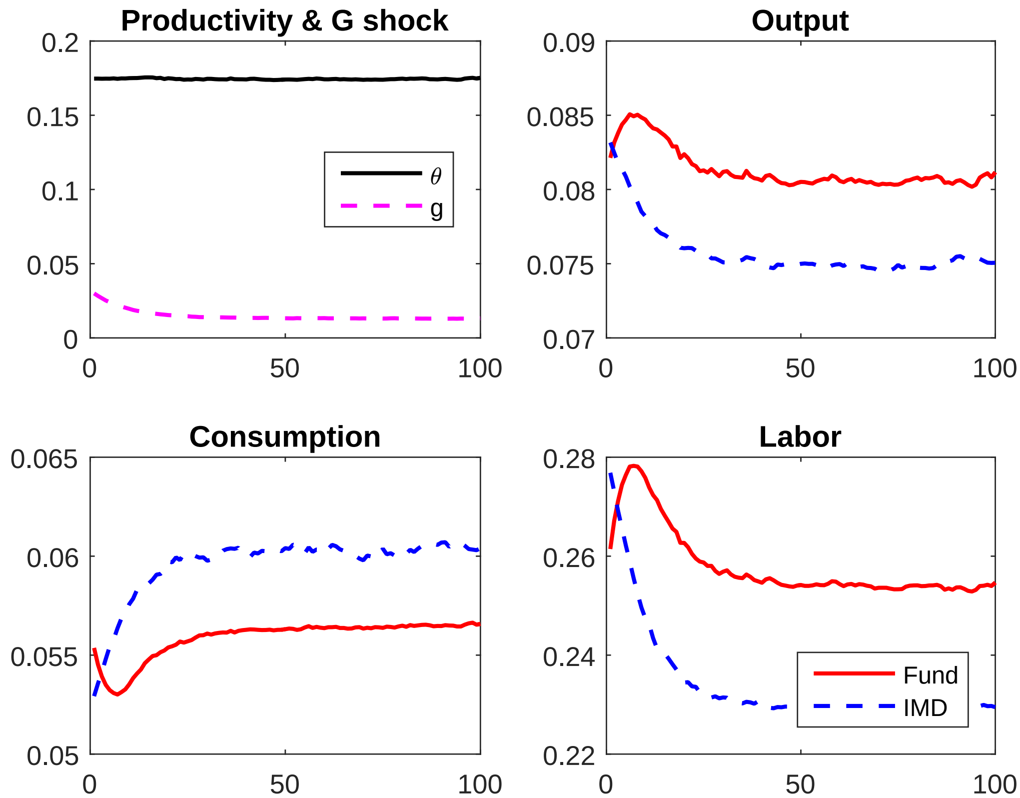

Figure 12: IMD vs. FSF: G shock impulse-responses: allocations.

$$c\,(x,s) = \frac{1+\upsilon_b(x,s)}{1+\upsilon_l(x,s)}\frac{\eta}{x},$$

$$c\,(x,s)\,\gamma\,(1-n\,(x,s))^{-\sigma} = \theta\alpha n\,(x,s)^{\alpha-1}\,,$$

$$x\,(s') = \frac{1+\upsilon_l(x,s)}{1+\upsilon_b(x,s)+\varphi(x,s')}\frac{x}{\eta},$$

where $\varphi(x,s')$ is given by

$$\varphi(x,s') = \tilde{\xi}(s)\frac{\rho\exp(-\rho e)\left[\pi^g(G'|G)-\pi^b(G'|G)\right]}{\pi^G(G'|G,e)}$$

and $\tilde{\xi}(s)$ is the Lagrange multiplier of the incentive constraint in the normalized problem; i.e. $\tilde{\xi}(s) = \frac{\xi(s)}{\mu_b(s)}$. Furthermore,

$$2\omega e = \beta\rho\exp(-\rho e)\sum_{G',\theta'}\left[\pi^g(G'|G)-\pi^b(G'|G)\right]\pi^\theta(\theta'|\theta)V^{bf}(x',s'),$$

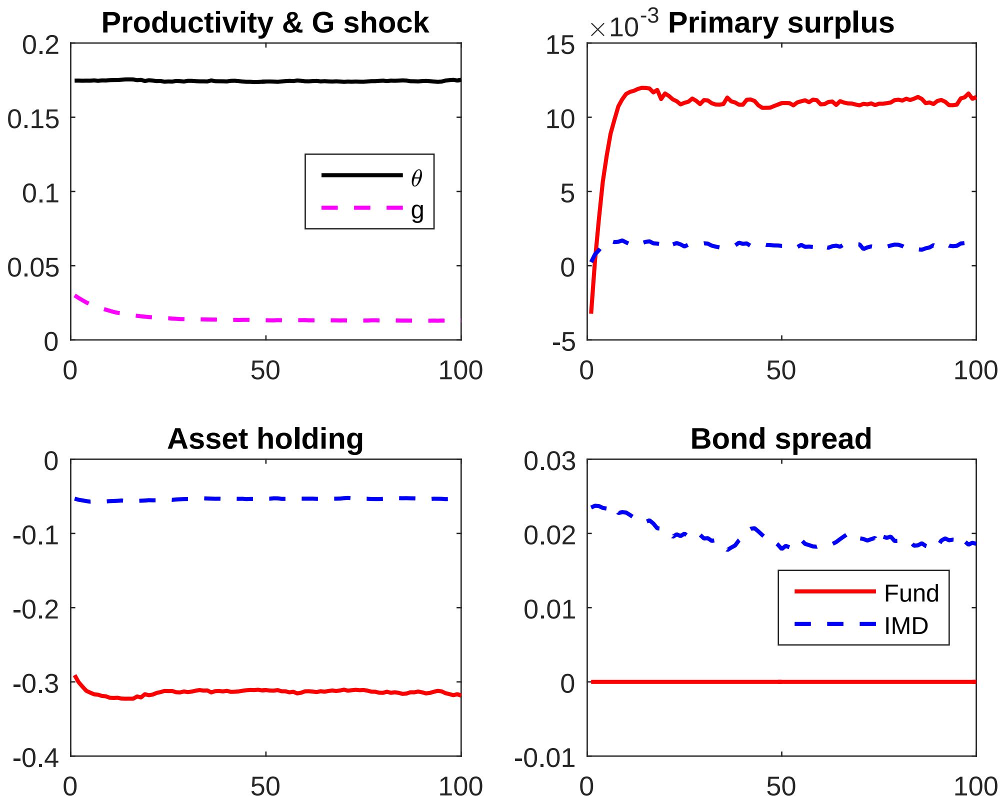

Figure 13: IMD vs. FSF: G shock impulse-responses: assets.

$$
\begin{aligned}
0 \quad = \quad & \rho \exp(-\rho e) \sum_{G',\theta'} \left[\pi^g(G'|G) - \pi^b(G'|G)\right] \pi^\theta(\theta'|\theta) \left[\frac{1}{1+r} V^{lf}\left(x\left(s'\right), s'\right) + \xi(x,s)\beta\rho V^{bf}(x(s'), s')\right] \\
& -\xi(x,s) 2\omega
\end{aligned}
$$

$$
V^{bf}(x,s) = \log(c(x,s)) + \frac{\gamma(1-n(x,s))^{1-\sigma}}{1-\sigma} - \omega e^2 + \beta \sum_{s' \in S} \pi^G(G'|G,e)\pi^\theta(\theta'|\theta) V^{bf}(x(s'), s').
$$

$$
V^{lf}(x,s) = \theta n(x,s)^\alpha - c(x,s) - G + \frac{1}{1+r} \sum_{s' \in S} \pi^G(G'|G,e)\pi^\theta(\theta'|\theta) V^{lf}(x(s'), s').
$$

$$
V^{af}(s) = \max_n \left\{ \begin{array}{c} \log(\theta n^\alpha - (1-\phi)G) + \frac{\gamma(1-n)^{1-\sigma}}{1-\sigma} - \omega e^2 + \\ +\beta \sum_{s' \in S} \pi^G(G'|G,e)\pi^\theta(\theta'|\theta) V^{af}(s') \end{array} \right\}
$$

The solution to this system of equations is found numerically using a policy iteration algorithm. More precisely, we discretize the relative pareto weight for the borrower x. For each grid point, we can calculate the value of autarky by solving for the optimal labor in autarky first and calculating $V^{af}(s)$ from the previous equation.

We then define the region of pareto weights between which none of the participation constraints are binding. In that region, for each shock $s = (\theta, G)$, the solution is characterized by the first best:

$$
\begin{aligned}
\upsilon_b(x,s) &= \upsilon_l(x,s) = 0 \\
x' &= x/\eta \\
c(x,s) &= \eta/x \text{ and } c_l(x,s) = \theta n(x,s)^\alpha - G - \eta/x \\
\frac{\eta}{x}\gamma(1-n(x,s))^{-\sigma} &= \theta\alpha n(x,s)^{\alpha-1} \\
V^{lf}(x,s) &= \theta n(x,s)^\alpha - G - \eta x + \frac{1}{1+r}\sum_{s'\in S}\pi(s'|s)V^{lf}(x',s') \\
V^{bf}(x,s) &= \log(\eta/x) + \frac{\gamma(1-n(x,s))^{1-\sigma}}{1-\sigma} + \beta\sum_{s'\in S}\pi(s'|s)V^{bf}(x',s').
\end{aligned}
$$

To find the region for which the participation constraint binds for the borrower, for each shock $s = (\theta, G)$, we find $c(x_b, s) = \eta/x_b$ such that $V^{bf}(x_b, s) = V^{af}(s)$. For the decentralization, using one-period Arrow securities, the bond price simplifies to:

$$
\begin{aligned}
q(s'|s) &= \max\left\{\beta\pi(s'|s)\frac{u'(c(x',s'))}{u'(c(x,s))}, \left(\frac{1}{1+r}\right)\pi(s'|s)\right\} \\
&= \pi(s'|s)\max\left\{\beta\frac{c(x,s)}{c(x',s')}, \left(\frac{1}{1+r}\right)\right\}
\end{aligned}
$$

The price of a one period bond is then equal to:

$$
q^f(s) = \sum_{s'\in S} q(s'|s)
$$

which in turn implies a risk free rate of $r^f(s) = \frac{1}{q^f(s)}$. Finally, we can recover the asset holdings numerically by iterating to find the asset holding function that satisfies:

$$
\begin{aligned}
a_b(x,s) &= \sum_{\theta'\in S} q(s')\,a_b(x',s') + c(x,s) - \theta f(n(x,s)) + G \\
a_l(x,s) &= -a_b(x,s)
\end{aligned}
$$

Moreover, we define the repayment as:

$$
a_b(x',s') - \sum_{s'\in S} q(s'|s)\,a_b(x',s')
$$

8.4 Further notes on the calibration procedure

On the transition matrix of the G shock. Note that this specification of the transition matrix is motivated by the one-period-crash Markov chain of Rietz (1988):

$$
\pi^R = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \eta & \phi & 1-\phi-\eta \\ \eta & 1-\phi-\eta & \phi \end{bmatrix}
$$

where the first state is labeled as the "crash" or "crisis" state, and the associated stationary distribution is

$$
\mu^R = \begin{bmatrix} \frac{\eta}{1+\eta} & \frac{1}{2(1+\eta)} & \frac{1}{2(1+\eta)} \end{bmatrix}
$$

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